GATE – 2021

MECHANICAL ENGINEERING (Morning)

GENERAL APTITUDE (GA)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

1. Consider the following sentences:

(i) After his surgery, Raja hardly could walk.

(ii) After his surgery, Raja could barely walk.

(iii) After his surgery, Raja barely could walk

(iv) After his surgery, Raja could hardly walk.

Which of the above sentences are grammatically CORRECT?

(a) (i) and (ii)

(b) (i) and (iii)

(c) (iii) and (iv)

(d) (ii) and (iv)

Ans. d

Exp:

Hardly/Scarcely/Barely have same sense that is negative and they are used after the verb (could barely and could hardly).

2. Ms. X came out of a building through its front door to find her shadow due to the morning sun falling to her right side with the building to her back. From this, it can be inferred that building is facing _________

(a) North

(b) East

(c) West

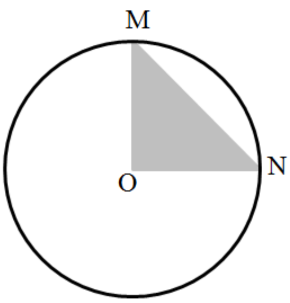
(d) South

Ans. d

Exp:

Morning sun is falling from east then the shadow will fall to the west. So, west should be on the right side of Ms. X. So, Ms. X came out towards south. Hence, her building is facing south.

3. In the above figure, O is the centre of the circle and, M and N lie on the circle. The area of the right triangle MON is 50 cm^2. What is the area of the circle in cm^2 ?



(a) 2π

(b) 50π

(c) 75π

(d) 100π

Ans. d

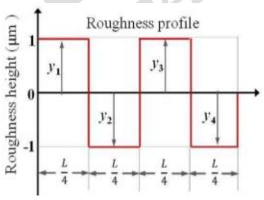
Exp:

∵ Area of ΔMON = 50 cm^2

$$\frac{1}{2} \times r^2 = 50$$

Area of circle = πr^2 = 100π

4.



Then, the value of the expression Δ 2 ⊕ 3 Δ ((4 ⊗ 2) ∇ 4) =

(a) –1

(b) –0.5

(c) 6

(d) 7

Ans. d

Exp:

$\Delta\, 2 \oplus 3\, \Delta\, ((4 \otimes 2)\, \nabla\, 4) = +2 - 3 + ((4 \div 2) \times 4) = +2 - 3 + (2 \times 4) = 7$

5. "The increased consumption of leafy vegetables in the recent months is a clear indication that the people in the state have begun to lead a healthy lifestyle"

Which of the following can be logically inferred from the information presented in the above statement?

(a) The people in the state did not consume leafy vegetables earlier.

(b) Consumption of leafy vegetables may not be the only indicator of healthy lifestyle.

(c) Leading a healthy lifestyle is related to a diet with leafy vegetables.

(d) The people in the state have increased awareness of health hazards causing by consumption of junk foods.

Ans. c

Exp:

The last sentence of the passage is reflecting in option c only.

Q.6 – Q.10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: –2/3).

6. Oxpeckers and rhinos manifest a symbiotic relationship in the wild. The oxpeckers warn the rhinos about approaching poachers, thus possibly saving the lives of the rhinos. Oxpeckers also feed on the parasitic ticks found on rhinos.

In the symbiotic relationship described above, the primary benefits for oxpeckers and rhinos respectively are,

(a) Oxpeckers get a food source, rhinos have no benefit.

(b) Oxpeckers save their habitat from poachers while the rhinos have no benefit.

(c) Oxpeckers get a food source, rhinos may be saved from the poachers.

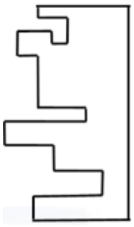
(d) Oxpeckers save the lives of poachers, rhinos save their own lives.

Ans. c

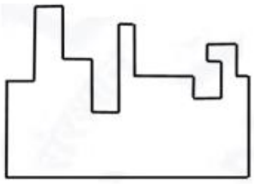
Exp:

Option (a) and (b) are weekend by expression 'rhinos have no benefit'. Oxpeckers do not save life of poachers.

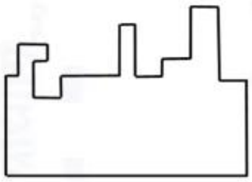
7. A jigsaw puzzle has 2 pieces. One of the pieces is shown below. Which one of the given options for the missing piece when assembled will form a rectangle? The piece can be moved, rotated or flipped to assemble with the above piece.



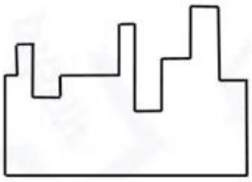
(a)



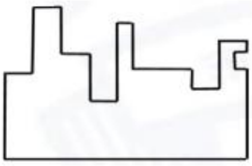
(b)



(c)



(d)



Ans. a

8. The number of hens, ducks and goats in farm P are 65, 91 and 169, respectively. The total number of hens, ducks and goats in a nearby farm Q is 416. The ratio of hens : ducks : goats in farm Q is 5 : 14 : 13. All the hens, ducks and goats are sent from farm Q to farm P.

The new ratio of hens : ducks : goats in farm P is ________.

(a) 5 : 7 : 13

(b) 5 : 14 : 13

(c) 10 : 21 : 26

(d) 21 : 10 : 26

Ans. c

Exp:

In farm P,

Hens = 65, Ducks = 91, Goats = 169

In farm Q,

Hens : Ducks : Goats

5 : 14 : 13

Hens = $\frac{5}{32} \times 416 = 65$

Ducks = $\frac{14}{32} \times 416 = 182$

Goats = $\frac{13}{32} \times 416 = 169$

$\because$ From farm d, hens, ducks and goats are sent to farm P,

$\therefore$ Total hens = 65 + 65 = 130

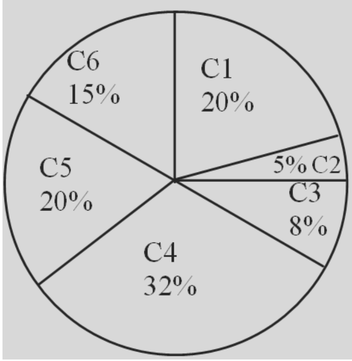
Total ducks = 91 + 182 = 273

Total goats = 169 + 169 = 338

New ratio = 130 : 273 : 338 = 10 : 21 : 26

9. The distribution of employees at the rank of executives, across different companies C1, C2, ..., C6 is presented in the chart given above. The ratio of executives with a management degree to those without a management degree in each of these companies is provided in the table above. The total number of executives across all companies is 10,000.

The total number of management degree holders among the executives in companies C2 and C5 together is_______.



Company	Ratio
C1	3 : 2
C2	1 : 4
C3	5 : 3
C4	2 : 3
C5	9 : 1
C6	3 : 4

(a) 225

(b) 600

(c) 1900

(d) 2500

Ans. c

Exp:

Number of employee in C2 company = $\frac{5}{100} \times 10000 = 500$

Number of management degree holder employee in C2 = $\frac{1}{5} \times 500 = 100$

Number of employee in C5 company = $\frac{20}{100} \times 10000 = 2000$

Number of management degree holder employee in C5 = $\frac{9}{10} \times 2000 = 1800$

Total management degree holder employee = 100 + 1800 = 1900

10. Five persons P, Q, R, S and T are sitting in a row not necessarily in the same order. Q and R are separated by one person, and S should not be seated adjacent to Q.

The number of distinct seating arrangements possible is:

(a) 4

(b) 8

(c) 10

(d) 16

Ans. d

Exp:

The possible seating arrangements are

QPRST, QTRSP, QPRTS, QTRPS, PQTRS, TQPRS, SPQTR, STQPR, RPQTS, RTQPS, SRPQT, SRTQP, SPRTQ, STRPQ, PSRTQ, TSRPQ

Hence, total seating arrangements are 16.

MECHANICAL ENGINEERING (MORNING)

Q.1 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

1. If y(x) satisfies the differential equation

$$(\sin x)\frac{dy}{dx} + y\cos x = 1,$$

subject to the condition y(π/2) = π/2, then y(π/6) is

(a) 0

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

Ans. c

Exp:

$$\frac{dy}{dx} + y\cot x = \text{cosec}\, x$$

Integrating factor, I.F. = $e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$

⇒ $y(\sin x) = \int \text{cosec}\, x \sin x dx + c$

⇒ $y\sin x = x + c$

⇒ $\frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2} + c$

⇒ c = 0

⇒ $y\sin x = x$

⇒ $y\sin\frac{\pi}{6} = \frac{\pi}{6}$

⇒ $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\Rightarrow \qquad y = \frac{\pi}{3}$

2. The value of $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ is

(a) $\frac{1}{4}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 1

Ans. c

Exp:

$\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ This is $\frac{0}{0}$ form.

Applying L-Hospital rule, we get $\frac{1}{2}$.

3. The Dirac-delta function ($\delta(t - t_o)$) for t, $t_o \in \mathbb{R}$, has the following property

$$\int_a^b \varphi(t)\delta(t-t_o)\,dt = \begin{cases} \varphi(t_o) & a < t_o < b \\ 0 & otherwise \end{cases}$$

The Laplace transform of the Dirac-delta function $\delta(t - a)$ for $a > 0$; $\mathcal{L}(\delta(t - a)) = F(s)$ is

(a) 0

(b) ∞

(c) e^{sa}

(d) e^{-sa}

Ans. d

Exp:

$\because \quad \int_0^\infty f(t)\delta(t-a)\,dt = f(a)$

$\therefore \quad L\{\delta(t-a)\} = \int_0^\infty e^{-st}\delta(t-a)\,dt = e^{-as}$

4. The ordinary differential equation $\frac{dy}{dx} = -\pi y$ subject to an initial condition y(0) = 1 is solved numerically using the following scheme:

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

where h is the time step, t_n = nh, and n = 0, 1, 2, ... This numerical scheme is stable for all values of h in the interval ________

(a) $0 < h < \frac{2}{\pi}$

(b) 0 < h < 1

(c) $0 < h < \frac{\pi}{2}$

(d) for all h > 0

Ans. a

Exp:

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

$$y_{n+1} = -\pi h y_n + y_n = (-\pi h + 1) y_n$$

It is recursion relation between y_{n+1} and y_n so the solution will be stable if

$|-\pi h + 1| < 1$

$-1 < -\pi h + 1 < 1$

$-2 < -\pi h < 0$

$0 < \pi h < 2$

$0 < h < \frac{2}{\pi}$

5. Consider a binomial random variable X. If $X_1, X_2, \ldots, X_n$ are independent and identically distributed samples from the distribution of X with sum Y = $\sum_{i=1}^{n} X_i$, then the distribution of Y as n $\rightarrow \infty$ can be approximated as

(a) Exponential

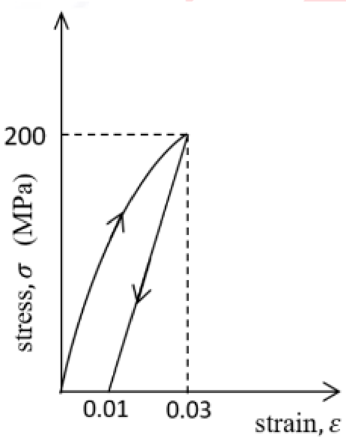
(b) Bernoulli

(c) Binomial

(d) Normal

Ans. d

6. The loading and unloading response of a metal is shown in the figure. The elastic and plastic strains corresponding to 200 MPa stress, respectively, are



(a) 0.01 and 0.01

(b) 0.02 and 0.01

(c) 0.01 and 0.02

(d) 0.02 and 0.02

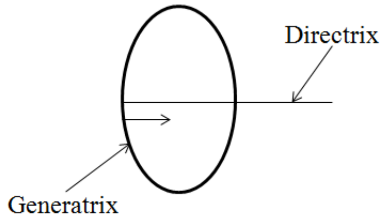
Ans. b

Exp:

Elastic strain : Which can be recovered = 0.03 – 0.01 = 0.02

Plastic strain : Permanent strain = 0.01

7. In a machining operation, if a cutting tool traces the workpiece such that the directrix is perpendicular to the plane of the generatrix as shown in figure, the surface generated is



(a) plane

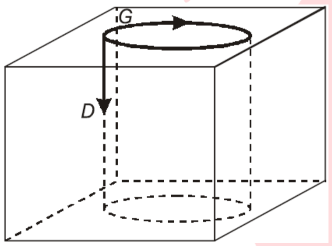
(b) cylindrical

(c) spherical

(d) a surface of revolution

Ans. b

Exp:



8. The correct sequence of machining operations to be performed to finish a large diameter through hole is

(a) drilling, boring, reaming

(b) boring, drilling, reaming

(c) drilling, reaming, boring

(d) boring, reaming, drilling

Ans. a

9. In modern CNC machine tools, the backlash has been eliminated by

(a) preloaded ballscrews

(b) rack and pinion

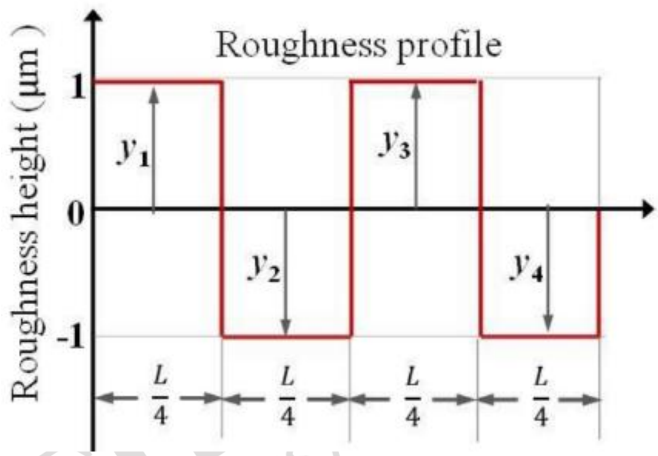
(c) ratchet and pinion

(d) slider crank mechanism

Ans. a

Exp:

10. Consider the surface roughness profile as shown in the figure :



The center line average roughness (Ra, in μm) of the measured length (L) is

(a) 0

(b) 1

(c) 2

(d) 4

Ans. b

Exp:

$$R_G = \frac{\sum_{i=1}^{n} y}{n} = \frac{4}{4} = 1$$

11. In which of the following pairs of cycles, both cycles have at least one isothermal process?

(a) Diesel cycle and Otto cycle

(b) Carnot cycle and Stirling cycle

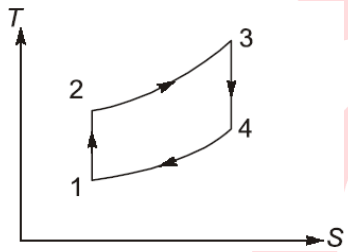
(c) Brayton cycle and Rankine cycle

(d) Bell-Coleman cycle and Vapour compression refrigeration cycle

Ans. b

Exp:

1. Brayton



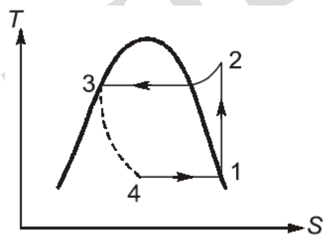
1 to 2 : S = C

2 to 3 : P = C

3 to 4 : S = C

4 to 1 : P = C

2. VCRS



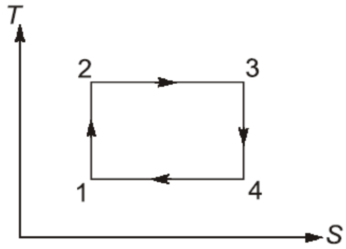
1 to 2 : Isentropic compression

2 to 3 : constant pressure (P = C)

3 to 4 : Isenthalpic expansion ($h_3 = h_4$)

4 to 1 : Constant pressure (P = C)

3. Carnot



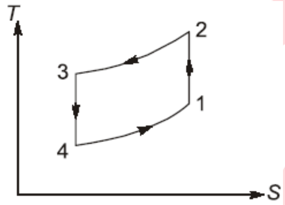
1 to 2 : Isentropic compression

2 to 3 : Isothermal heat addition

3 to 4 : Isentropic expansion

4 to 1 : Isothermal heat rejection

4. Bell Coleman



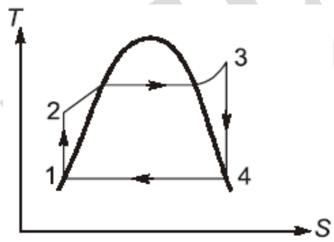
1 to 2 : Isentropic compression

2 to 3 : Isothermal heat addition

3 to 4 : Isentropic expansion

4 to 1 : Isothermal heat rejection

5. Rankine



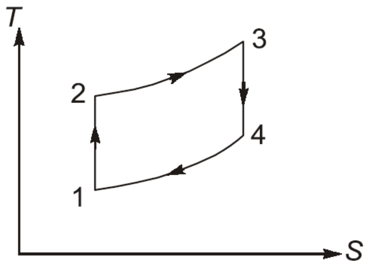
1 to 2 : Isentropic compression

2 to 3 : Constant pressure

3 to 4 : Isentropic expansion

4 to 1 : Constant pressure

6. Diesel



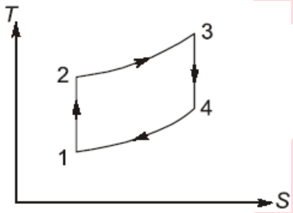
1 to 2 : Isentropic compressin

2 to 3 : Constant pressure (P = C)

3 to 4 : Isentropic expansion

4 to 1 : Constant volume

7. Otto



1 to 2 : Isentropic compression

2 to 3 : Constant pressure (V = C)

3 to 4 : Isentropic expansion

4 to 1 : Constant volume (V = C)

12. Superheated steam at 1500 kPa, has a specific volume of 2.75 m^3/kmol and compressibility factor (Z) of 0.95. The temperature of steam is______ºC (round off to the nearest integer).

(a) 522

(b) 471

(c) 249

(d) 198

Ans. c

Exp:

P = 1500 kPa, V = 2.75 m^3/k-mol, Z = 0.95

PV = n $\bar{R}$ T

$P\bar{V} = \bar{R}T = Z \times n\bar{R}T$

$$P\frac{\bar{V}}{n} = Z\bar{R}T$$

$$P\bar{V} = Z\bar{R}T$$

1500 kPa × 2.75 m^3/k-mol = 0.95 × 8.314 kJ/k-molK × T

T = 522.26 K = 249.26°C

13. A hot steel spherical ball is suddenly dipped into a low temperature oil bath. Which of the following dimensionless parameters are required to determine instantaneous center temperature of the ball using a Heisler chart?

(a) Biot number and Fourier number

(b) Reynolds number and Prandtl number

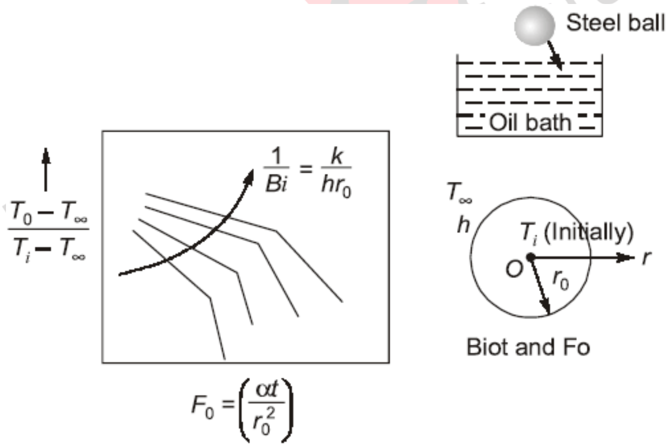
(c) Biot number and Froude number

(d) Nusselt number and Grashoff number

Ans. a

Exp:

Unsteady state



14. An infinitely long pin fin, attached to an isothermal hot surface, transfers heat at a steady rate of $\dot{Q}_1$ to the ambient air. If the thermal conductivity of the fin material is doubled, while keeping everything else constant, the rate of steady - state heat transfer from the fin becomes $\dot{Q}_2$. The ratio $\dot{Q}_2/\dot{Q}_1$ is

(a) $\sqrt{2}$

(b) 2

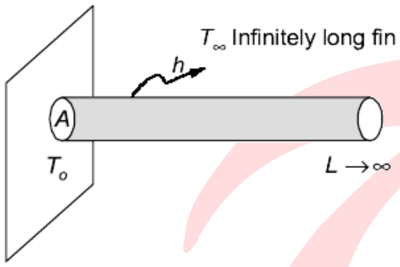
(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{2}$

Ans. a

Exp:

Fin problem:



$q = \sqrt{hPkA}\left(T_o - T_\infty\right)$ wait

If k gets doubled q increases by $\sqrt{2}$ times.

15. The relative humidity of ambient air at 300 K is 50% with a partial pressure of water vapour equal to p_v. The saturation pressure of water at 300 K is p_{sat}. The correct relation for the air-water mixture is

(a) $p_v = 0.5\ p_{sat}$

(b) $p_v = p_{sat}$

(c) $p_v = 0.622\ p_{sat}$

(d) $p_v = 2\ p_{sat}$

Ans. a

Exp:

Relative humidity, $\phi = 0.5 = \frac{p_v}{p_{sat}}$

$p_v = 0.5\ p_{sat}$

16. Consider a reciprocating engine with crank radius R and connecting rod of length L. The secondary unbalance force for this case is equivalent to primary unbalance force due to a virtual crank of ________.

(a) radius $\frac{L^2}{4R}$ rotating at half the engine speed

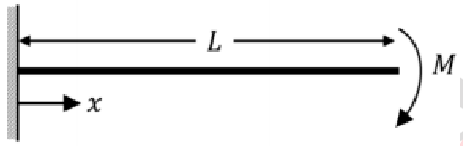
(b) radius $\frac{R}{4}$ rotating at half the engine speed

(c) radius $\frac{R^2}{4L}$ rotating at twice the engine speed

(d) radius $\frac{L}{2}$ rotating at twice the engine speed

Ans. c

17. A cantilever beam of length , L and flexural rigidity, EI is subjected to an end moment, M, as shown in figure. The deflection of the beam at $x = \frac{L}{2}$ is



(a) $\frac{ML^2}{2EI}$

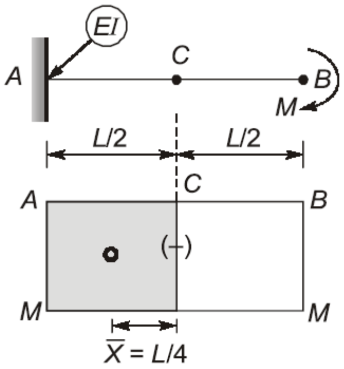
(b) $\frac{ML^2}{4EI}$

(c) $\frac{ML^2}{8EI}$

(d) $\frac{ML^2}{16EI}$

Ans. c

Exp:



$$Y_C - Y_A = \left(\frac{A\overline{X}}{EI}\right)_{AC}$$

$$Y_C - 0 = \frac{1}{EI}\left[\frac{-ML}{2} \times \frac{L}{4}\right]$$

$$Y_C = \frac{ML^2}{8EI} \qquad (\downarrow)$$

18. A prismatic bar PQRST is subjected to axial loads as shown in the figure. The segments having maximum and minimum axial stresses, respectively, are

(a) QR and PQ

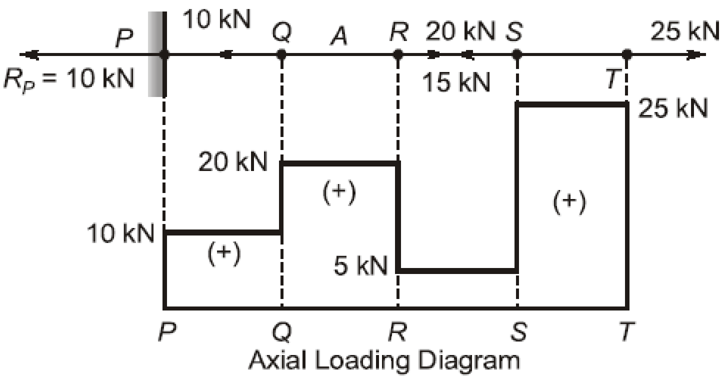
(b) ST and PQ

(c) QR and RS

(d) ST and RS

Ans. d

Exp:



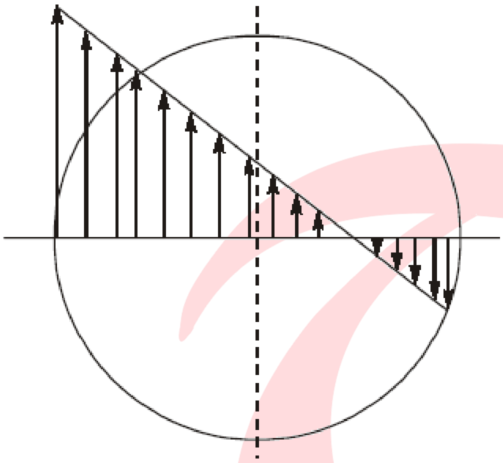
Axial Loading Diagram

$P_{max} = P_{ST} = 25$ kN

$P_{min} = P_{RS} = 5$ kN

Hence, maximum and minimum axial stresses are in ST and RS portions because of prismatic bar. Hence, the answer is (c).

19. Shear stress distribution on the cross-section of the coil wire in a helical compression spring is shown in the figure. This shear stress distribution represents



(a) direct shear stress in the coil wire cross-section

(b) torsional shear stress in the coil wire cross-section

(c) combined direct shear and torsional shear stress in the coil wire cross-section

(d) combined direct shear and torsional shear stress along with the effect of stress concentration at inside edge of the coil wire cross-section

Ans. c

Exp:

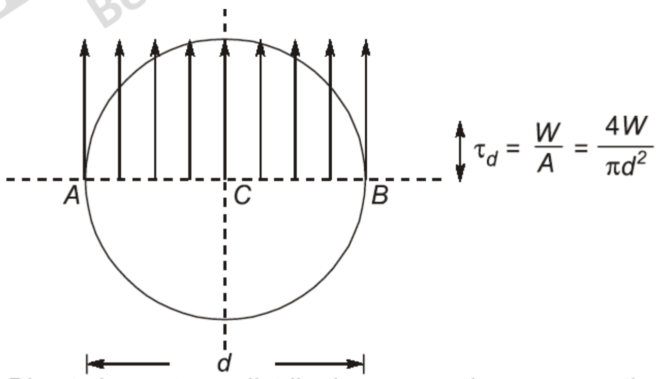


Fig. : Direct shear stress distribution across the cross-section of wire

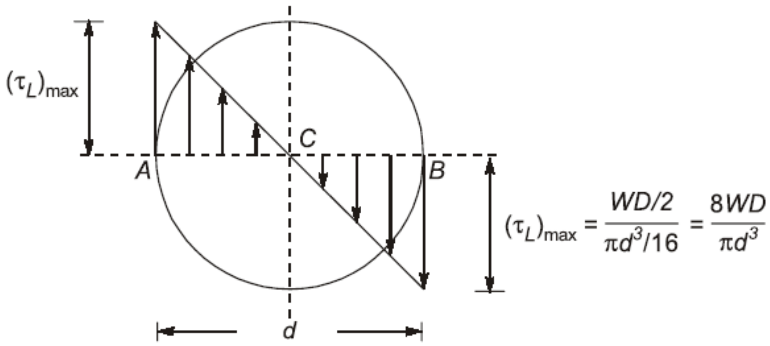


Fig. : Torsional shear stress distribution across the cross-section of wire

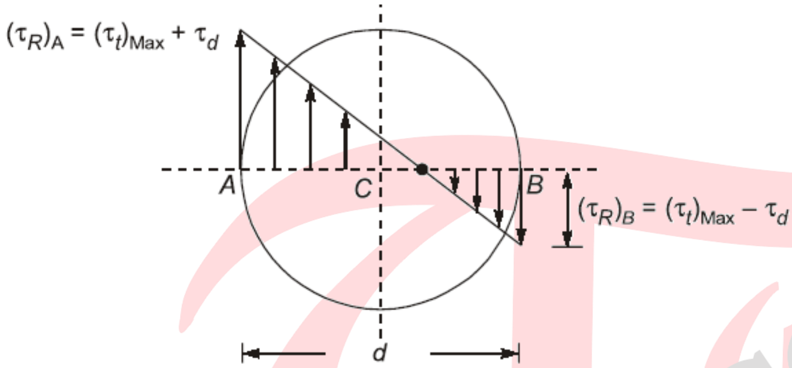


Fig. : Resultant shear stress variation across the cross-section of wire (without considering stress concentration effect)

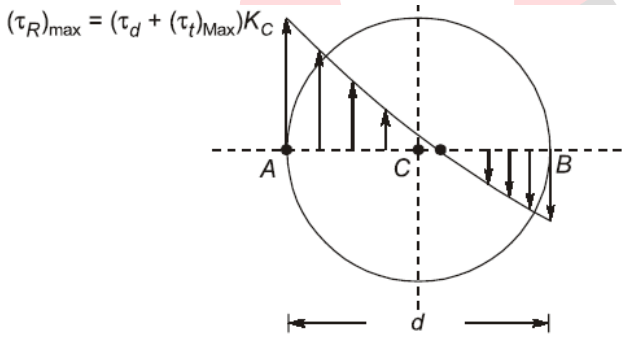


Fig. : Resultant shear stress variation considering stress concentration effect

Q.20 – Q.25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

20. Robot Ltd. wishes to maintain enough safety stock during the lead time period between starting a new production run and its completion such that the probability of satisfying the customer demand during the lead time period is 95%. The lead time period is 5 days and daily customer demand can be assumed to follow the Gaussian (normal) distribution with mean 50 units and a standard deviation of 10 units. Using φ^{-1} (0. 95) = 1. 64, where φ represents the cumulative distribution function of the standard normal random variable, the amount of safety stock that must be maintained by Robot Ltd. to achieve this demand fulfilment probability for the lead time period is ______________ units (round off to two decimal places).

Ans. 331.00 – 333.00

Exp:

Safety stock = Z × σ

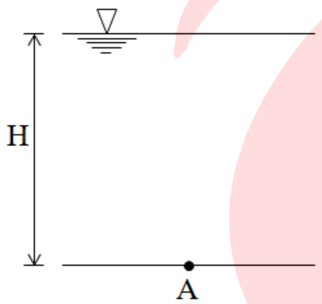
$$\sigma = \sqrt{10^2 + 10^2 + 10^2 + 10^2 + 10^2} = 22.3606$$

Safety stock = 1.64 × 22.3606 = 36.67

21. A pressure measurement device fitted on the surface of a submarine, located at a depth H below the surface of an ocean, reads an absolute pressure of 4.2 MPa. The density of sea water is 1050g/m^3, the atmospheric pressure is 101 kPa, and the acceleration due to gravity is 9.8 m/s2. The depth H is ___________ m (round off to the nearest integer).

Ans. 397 – 399

Exp:



P_A = 4.2 MPa [Absolute pressure]

P_{atm} = 101 kPa

ρ = 1050 kg/m^3

g = 9.8 m/s^2

$P_A = P_{atm} + \rho g H$

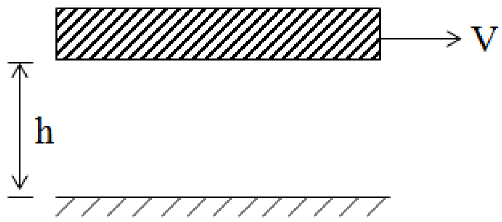
$4.2 \times 10^6 = (101 \times 10^3) + [1050 \times 9.81 \times H]$

H = 397.94 or 398 m

22. Consider fully developed, steady state incompressible laminar flow of a viscous fluid between two large parallel horizontal plates. The bottom plate is fixed and the top plate moves with a constant velocity of U = 4 m/s. Separation between the plates is 5 mm. There is no pressure gradient in the direction of flow. The density of fluid is 800 kg/m^3 , and the kinematic viscosity is 1.25×10^{-4} m^2/s. The average shear stress in the fluid is _________ Pa (round off to the nearest integer).

Ans. 79 – 81

Exp:



V = 4 m/s, ρ = 800 kg/m^3 , ν = 1.25 × 10^{-4} m^2/s, h = 5 mm

$$\tau = \mu \cdot \frac{du}{dy}$$

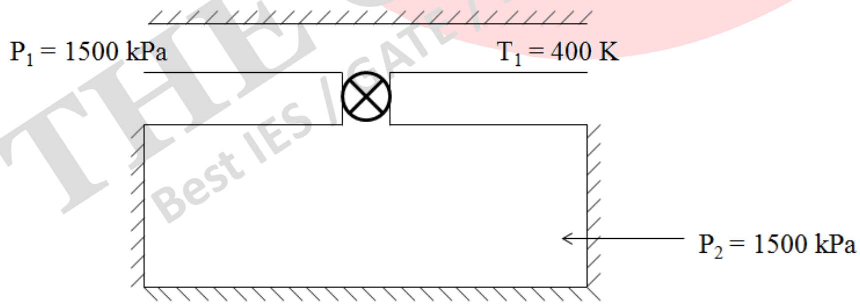
$$\tau = [800 \times 1.25 \times 10^{-4}] \times \frac{4}{5 \times 10^{-3}}$$

τ = 80 N/m^2 or 80 Pa

23. A rigid insulated tank is initially evacuated. It is connected through a valve to a supply line that carries air at a constant pressure and temperature of 250 kPa and 400 K respectively. Now the valve is opened and air is allowed to flow into the tank until the pressure inside the tank reaches to 250 kPa at which point the valve is closed. Assume that the air behaves as a perfect gas with constant properties (c_p = 1.005 kJ/kg.K, c_v = 0.718 kJ/kg.K, R = 0.287 kJ/kg.K). Final temperature of the air inside the tank is ___________ K (round off to one decimal place).

Ans. 555.0 – 565.0

Exp:

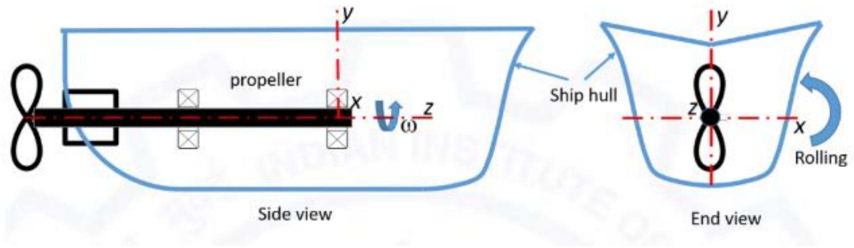


$$T_2 = \frac{C_P}{C_V} T_1$$

$$T_2 - \left(\frac{1.005}{0.718}\right) \times 400$$

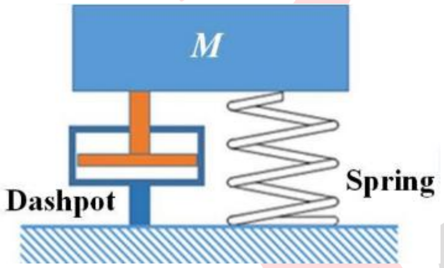
T_2 = 559.8 K

24. The figure shows an arrangement of a heavy propeller shaft in a ship. The combined polar mass moment of inertia of the propeller and the shaft is 100 kg.m^2 . The propeller rotates at ω = 12 rad/s. The waves acting on the ship hull induces a rolling motion as shown in the figure with an angular velocity of 5 rad/s. The gyroscopic moment generated on the shaft due to the motion described is ______________ N.m (round off to the nearest integer).



Ans. 0

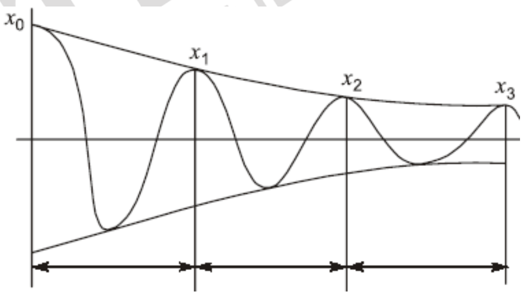
25. Consider a single degree of freedom system comprising a mass M, supported on a spring and a dashpot as shown in the figure.



If the amplitude of the free vibration response reduces from 8 mm to 1.5 mm in 3 cycles, the damping ratio of the system is ______________ (round off to three decimal places).

Ans. 0.085 – 0.090

Exp:



$$\frac{x_o}{x_3} = \frac{8}{1.5} = \frac{80}{15}$$

$$\frac{x_o}{x_1} \cdot \frac{x_1}{x_2} \cdot \frac{x_2}{x_3} = \frac{80}{15}$$

$e^{\delta} \cdot e^{\delta} \cdot e^{\delta} = \dfrac{80}{15}$

$\delta = 0.55799$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.55799$$

$\xi = 0.088$

Q.26 – Q.34 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: –2/3).

26. Consider a vector p in 2-dimensional space. Let its direction (counter-clockwise angle with the positive x-axis) be θ. Let p be an eigenvector of a 2 × 2 matrix A with corresponding eigenvalue λ, λ > 0. If we denote the magnitude of a vector v by ‖v‖, identify the VALID statement regarding p′, where p′ = Ap.

(a) Direction of p′ = λθ, ‖p′‖ = ‖p‖

(b) Direction of p′ = θ, ‖p′‖ = λ‖p‖

(c) Direction of p′ = λθ, ‖p′‖ = λ‖p‖

(d) Direction of p′ = θ, ‖p′‖ = ‖p‖/λ

Ans. b

Exp:

∵ A is a 2 × 2 matrix and P is the eigen vector of matrix A with corresponding eigen value λ.

Given: P' = P

AP = λP

Hence, P' = λP

‖P'‖ = ‖λP‖= λ‖P‖

But direction of vector P' will be same as vector P.

27. Let C represent the unit circle centered at origin in the complex plane, and complex variable, z = x + iy. The value of the contour integral $\oint_C \frac{\cosh 3z}{2z} dz$ (where integration is taken counter clockwise) is

(A) 0

(B) 2

(C) πi

(D) 2πi

Ans. c

Exp:

Pole of f(c) is z = 0 simple pole.

Reduce at z = 0,

$$\underset{\tau=0}{R_b}\, f(z) = \lim_{z\to 0}(z-0)\, f(z) = \lim_{z\to 0}\frac{\cosh(3z)}{2} = \lim_{z\to 0}\frac{e^{3Z}+e^{-3Z}}{2\times 2} = \frac{1}{2}$$

By Cauchy Riemann theorem

$$I = 2\pi\left(\frac{1}{2}\right) = \pi i$$

28. A set of jobs A, B, C, D, E, F, G, H arrive at time t = 0 for processing on turning and grinding machines. Each job needs to be processed in sequence – first on the turning machine and second on the grinding machine, and the grinding must occur immediately after turning. The processing times of the jobs are given below.

Job	A	B	C	D	E	F	G	H
Turning (minutes)	2	4	8	9	7	6	5	10
Grinding (minutes)	6	1	3	7	9	5	2	4

If the makespan is to be minimized, then the optimal sequence in which these jobs must be processed on the turning and grinding machines is

(a) A-E-D-F-H-C-G-B

(b) A-D-E-F-H-C-G-B

(c) G-E-D-F-H-C-A-B

(d) B-G-C-H-F-D-E-A

Ans. a

Exp:

Sequencing,

A	(2)	6
B	4	(1)
C	8	(3)
D	9	(7)
E	(7)	9
F	6	(5)

G	5	②
H	10	④

A E D F H C G B

29. The fundamental thermodynamic relation for a rubber band is given by dU = TdS + τdL, where T is the absolute temperature, S is the entropy, τ is the tension in the rubber band, and L is the length of the rubber band. Which one of the following relations is CORRECT:

(a) $\tau = \left(\frac{\partial U}{\partial S}\right)_L$

(b) $\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial S}\right)_L$

(c) $\left(\frac{\partial T}{\partial S}\right)_L = \left(\frac{\partial \tau}{\partial L}\right)_S$

(d) $T = \left(\frac{\partial U}{\partial S}\right)_\tau$

Ans. b

Exp:

dU = Tds + τdL

$$\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial \tau}{\partial S}\right)_L$$

Comparing with M = $\left(\frac{\partial z}{\partial x}\right)_y$

$$T = \left(\frac{\partial U}{\partial S}\right)_L$$

Comparing with N = $\left(\frac{\partial z}{\partial y}\right)_x$

$$\tau = \left(\frac{\partial U}{\partial L}\right)_S$$

$$M = \left(\frac{\partial z}{\partial x}\right)_y$$

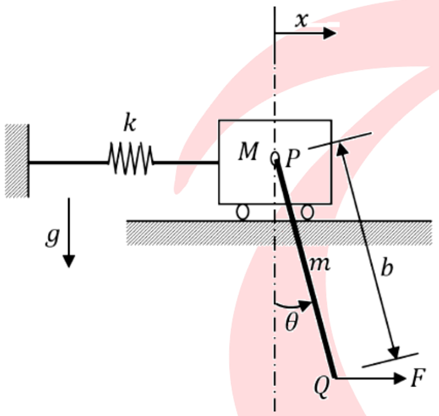
$$N = \left(\frac{\partial z}{\partial y}\right)_x$$

dz = Mdx + Ndy

If z is exact different

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

30. Consider a two degree of freedom system as shown in the figure, where PQ is a rigid uniform rod of length, b and mass, m.



Assume that the spring deflects only horizontally and force F is applied horizontally at Q. For this system, the Lagrangian, L is

(a) $\frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta$

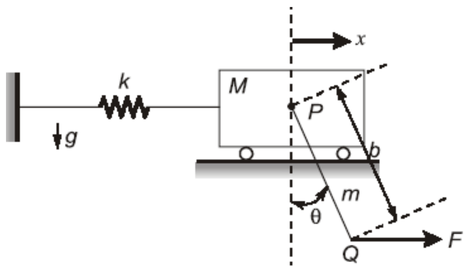
(b) $\frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta$

(c) $\frac{1}{2}M\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2$

(d) $\frac{1}{2}M\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{1}{6}mb^2\dot{\theta}^2 - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta + Fb\sin\theta$

Ans. b

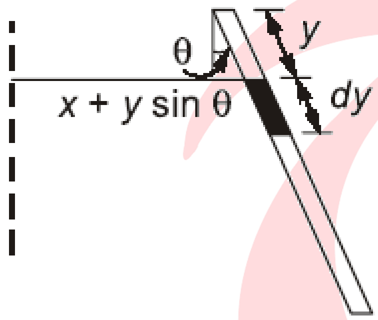
Exp:



For mass M, $T = \frac{1}{2} M\dot{x}^2$

$$V = \frac{1}{2} Kx^2$$

For mass m,



$$dm = \frac{m}{b} dy$$

Displacement = $x + y \sin\theta$

Velocity = $\dot{x} + \text{y} \cos\theta \, \dot{\theta}$

$$\text{dT} = \frac{1}{2} \text{dMVel}^2 = \frac{1}{2} \text{dm}\left[\dot{x}^2 + y^2\dot{\theta}^2 \cos^2\theta + 2\dot{x}\dot{\theta} y \cos\theta\right]$$

$$T = dT = \frac{1}{2}\frac{m}{b}\int\left(\dot{x}^2 + y^2\dot{\theta}^2 \times 1 + 2\dot{x}\dot{\theta} y \cos\theta\right) dy$$

$$T = \frac{1}{2}\frac{m}{b}\left(\dot{x}^2 b + \frac{b^3}{3}\dot{\theta}^2 + 2\dot{x}\dot{\theta}\cos\theta \frac{b^2}{2}\right)$$

$$T = \frac{1}{2}\frac{m}{b}\left(\dot{x}^2 b + \frac{b^3}{3}\dot{\theta}^2 + \dot{x}\dot{\theta}\text{b}^2 \cos\theta\right)$$

$$T = \frac{1}{2} m\dot{x}^2 + \frac{mb^2}{6}\dot{\theta}^2 + \frac{m}{2}\dot{x}\dot{\theta}\cos\theta$$

$$V = -mg\frac{b}{2}\cos\theta$$

For both masses, M and m

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{x}^2 + \frac{mb^2}{6}\dot{\theta}^2 + \frac{m}{2}b\dot{\theta}\dot{x}^2\cos\theta$$

$$V = \frac{1}{2}kx^2 - mg\frac{b}{2}\cos\theta$$

$$L = T - V = \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}mb\dot{\theta}\dot{x}\cos\theta + \frac{mb^2\dot{\theta}^2}{6} - \frac{1}{2}kx^2 + mg\frac{b}{2}\cos\theta$$

31. A right solid circular cone standing on its base on a horizontal surface is of height H and base radius R. The cone is made of a material with specific weight w and elastic modulus E. The vertical deflection at the mid-height of the cone due to self-weight is given by

(a) $\frac{wH^2}{8E}$

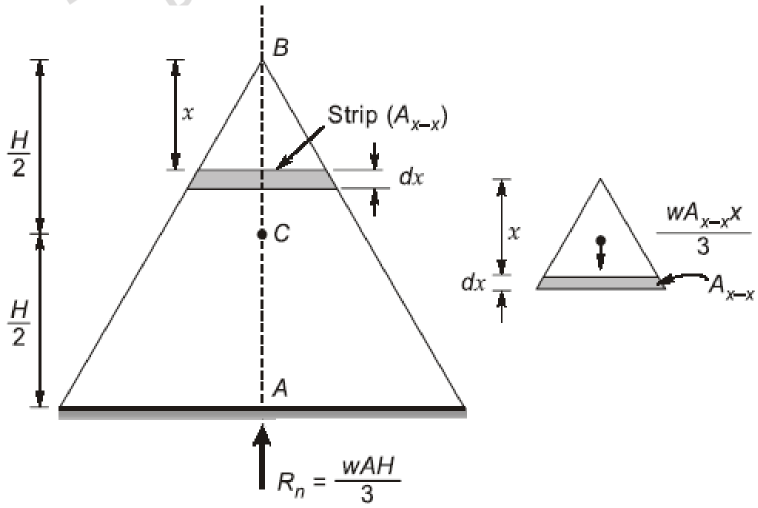
(b) $\frac{wH^2}{6E}$

(c) $\frac{wRH}{8E}$

(d) $\frac{wRH}{6E}$

Ans. a

Exp:



$$P_{x-x} = \frac{(-)\,wA_{x-x}(x)}{3}$$

Contraction of small strip = $(\delta l)_{strip}$

$$= \frac{P_{x-x}\,dx}{(AE)_{x-x}} = \frac{w(A_{x-x})(x)\,dx}{3(A_{x-x})E} = \frac{wx}{3E}dx$$

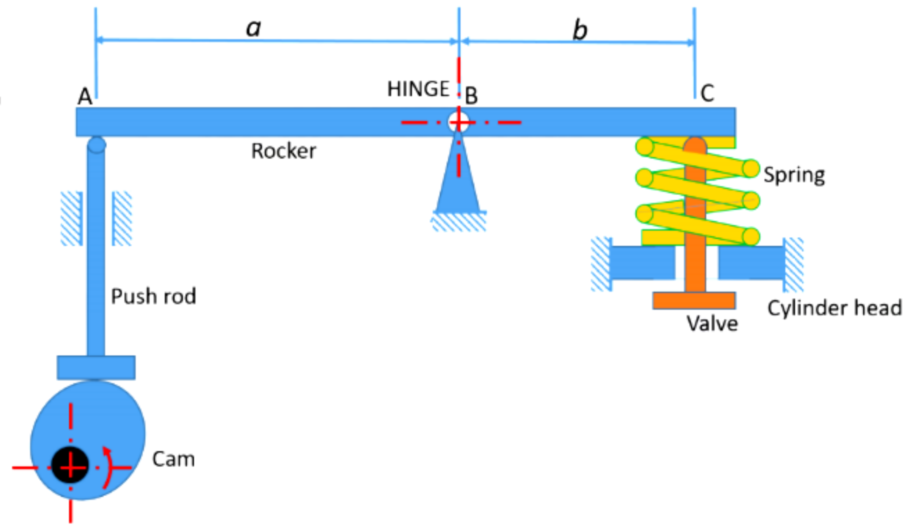
Contraction of conical bar at mid height $\left(\text{i.e., } x = \frac{H}{2}\right)$

$$= \int_{H/2}^{H} (\delta l)_{strip}$$

$$= \frac{w}{3E}\int_{H/2}^{H} x\,dx$$

$$= \frac{w}{6E}\left[H^2 - \frac{H^2}{4}\right] = \frac{wH^2}{8E}$$

32. A tappet valve mechanism in an IC engine comprises a rocker arm ABC that is hinged at B as shown in the figure. The rocker is assumed rigid and it oscillates about the hinge B. The mass moment of inertia of the rocker about B is 10^{-4} kg.m^2. The rocker arm dimensions are a = 3.5 cm and b = 2.5 cm. A pushrod pushes the rocker at location A, when moved vertically by a cam that rotates at N rpm. The pushrod is assumed massless and has a stiffness of 15 N/mm. At the other end C, the rocker pushes a valve against a spring of stiffness 10 N/mm. The valve is assumed massless and rigid.



Resonance in the rocker system occurs when the cam shaft runs at a speed of _________ rpm (round off to the nearest integer).

(a) 496

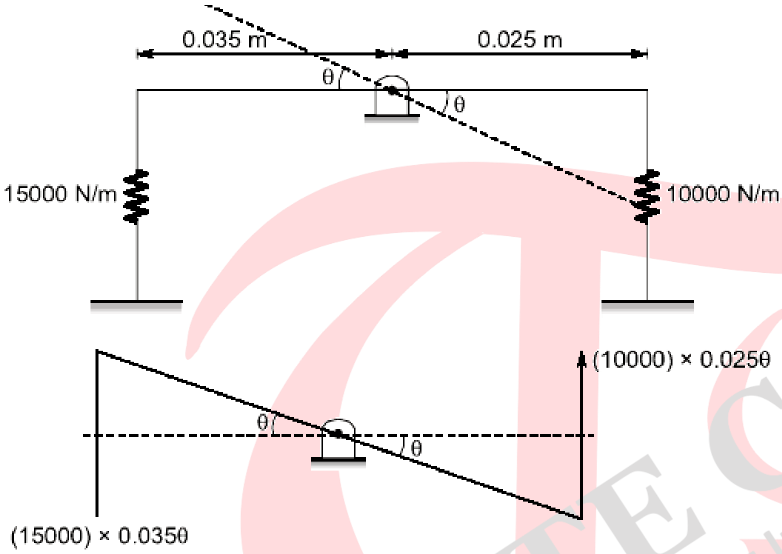
(b) 4739

(c) 790

(d) 2369

Ans. b

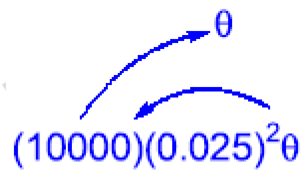
Exp:



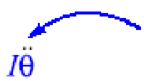
$I = 10^{-4}$ kg-m^2

By D'Alembert Principle

$I\ddot{\theta} + [10,000 \times (0.025)^2 + 15000 \times (0.035)^2]\theta = 0$



$(10^{-4})\ddot{\theta} + (24.625)\theta = 0$



$$\ddot{\theta} + \left(\frac{24.635}{10^{-4}}\right)\theta = 0$$

⇒ $\omega_n^2 = 246250$

ω_n = 496.2358 rad/s

⇒ $N_C = \dfrac{496.2358 \times 60}{2\pi}$ = 4738.7 ≃ 4739 rpm

33. Customers arrives at a shop according to the Poisson distribution with a mean of 10 customers/hour. The manager notes that no customer arrives for the first 3 minutes after the shop opens. The probability that a customer arrives within the next 3 minutes is

(a) 0.39

(b) 0.86

(c) 0.50

(d) 0.61

Ans. a

Exp:

(i) Probability of zero in 3 min ($\lambda T = 0.5$)

$$P_1 \to P(0,3) = \frac{(\exp)^{-0.5} \cdot (0.5)^0}{0!} = 0.606$$

(ii) Probability of 1 customer in 6 min (λ T = 1)

$$P_3 \to P(1,6) = \frac{(\exp)^{-1} \cdot (1)^1}{1!} = e^{-1} = 0.3678$$

$P_1 \times P_2 = P_3$

$0.606 \times P_2 = 0.3678$

$P_2 = 0.61$

34. Let f(x) = $x^2 - 2x + 2$ be a continuous function defined on $x \in [1, 3]$. The point x at which the tangent of f(x) becomes parallel to the straight line joining f(1) and f(3) is

(a) 0

(b) 1

(c) 2

(d) 3

Ans. c

Exp:

By Lagrangian mean value theorem,

$$\frac{f(3)-f(1)}{3-1}=f'(c)$$

$$\frac{5-1}{3-1}=2x-2$$

$x = 2 \quad \in \quad (1, 3)$

Q.35 – Q.55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).

35. Activities A, B, C and D form the critical path for a project with a PERT network. The means and variances of the activity duration for each activity are given below. All activity durations follow the Gaussian (normal) distribution, and are independent of each other.

Activity	A	B	C	D
Mean (days)	6	11	8	15
Variance (day^2)	4	9	4	9

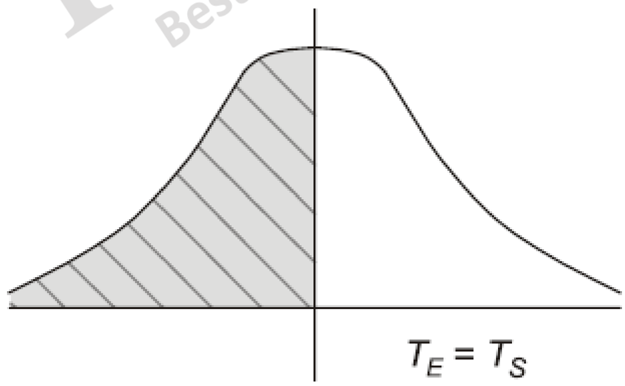
The probability that the project will be completed within 40 days is _________ (round off to two decimal places).

(Note: Probability is a number between 0 and 1).

Ans. 0.50

Exp:

PERT-CPM



T_S = 40 days, T_E = 6 + 11 + 8 + 15

T_E = 40 days ,

$$2 = \frac{T_S - T_E}{\sigma} = 0 \qquad \rightarrow \qquad 50\%$$

Probability of completing project in expected time is always 50% .

36. A true centrifugal casting operation needs to be performed horizontally to make copper tube sections with outer diameter of 250 mm and inner diameter of 230 mm. The value of acceleration due to gravity, g = 10 m/s^2 . If a G-factor (ratio of centrifugal force to weight) of 60 is used for casting the tube, the rotational speed required is ________________rpm (round off to the nearest integer).

Ans. 660 – 664

Exp:

Given : D_o = 250 mm, D_i = 230 mm, g = 10 m/s^2 , G-factor = 60

$F_c = mr\omega^2$

$F_g = mg$

G-factor = $\frac{F_c}{F_g} = 60$

G-factor = $\frac{F_c}{F_g} = \frac{mr\omega^2}{mg} = \frac{D_o}{2g}\left(\frac{2\pi N}{60}\right)^2$

$$N = \sqrt{\frac{2 \times G - Factor \times g}{D_o}} \times \left(\frac{60}{2\pi}\right) = \sqrt{\frac{2 \times 60 \times 10}{0.25}} \times \left(\frac{60}{2\pi}\right)$$

N = 661.93 = 662 rpm

37. The resistance spot welding of two 1.55 mm thick metal sheets is performed using welding current of 10000 A for 0.25 s. The contact resistance at the interface of the metal sheets is 0.0001 Ω. The volume of weld nugget formed after welding is 70 mm^3. Considering the heat required to melt unit volume of metal is 12 J/mm^3 , the thermal efficiency of the welding process is ________ % (round off to one decimal place).

Ans. 30 – 36

Exp:

Thickness = 1.55 mm

I = 10,000 A

t = 0.25 s

R = 0.0001 Ω

V_n = 70 mm^3

H_m = 12 J/mm^3

$$\eta_m = \frac{H_m}{H_s}$$

Melting efficiency = Thermal efficiency

$$\eta_m = \frac{H_m}{I^2RT} = \frac{12\times 70}{(10,000)^2\times 0.0001\times 0.25} = 0.336 = 33.6\ \%$$

38. An orthogonal cutting operation is performed using a single point cutting tool with a rake angle of 12° on a lathe. During turning, the cutting force and the friction force are 1000 N and 600 N, respectively. If the chip thickness and the uncut chip thickness during turning are 1.5 mm and 0.75 mm, respectively, then the shear force is ______________ N (round off to two decimal places).

Ans. 625 – 750

Exp:

F_c = 1000 N , F = 600 N, t = 0.75 mm, t_c = 1.5 mm, α = 12°

This is orthogonal turning operation

$$r = \frac{t}{t_c} = \frac{0.75}{1.5} = 0.5$$

$$\tan\phi = \frac{0.5\cos 12^o}{1-0.5\sin 12^o}$$

ϕ = 28.63°

Method – 1 : Using Merchant circle

$F_c = R\cos(\beta - \alpha)$

$F = R\sin\beta$

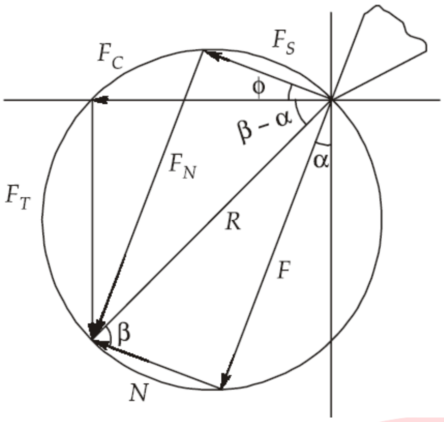
$$\frac{F}{F_c} = \frac{R\sin\beta}{R\cos(\beta-\alpha)}$$

$$\frac{600}{1000} = \frac{\sin\beta}{\cos(\beta-12)}$$

0.6 cos β cos 12° + 0.6 sin β sin 12° = sin β

$0.586\cos\beta + 0.1247\sin\beta = \sin\beta$

$0.586\cos\beta = 0.8753\sin\beta$



$\tan\beta = \dfrac{0.586}{0.8753}$

$\Rightarrow \quad \beta = 33.84°$

$F = R\sin\beta$

$\therefore \quad F = R\sin\beta$

$600 = R\sin 33.84°$

$R = 1077.44$ N

$\therefore \quad F_s = R\cos(\phi + \beta - \alpha)$

$= 1077.44\cos(28.63° + 33.84° - 12°) = 685.77$ N

Method – 2 : Using force relations

$F = F_c\sin\alpha + F_t\cos\beta$

$600 = 1000\sin 12° + F_t\cos 12°$

$F_t = 400.85$ N

$\therefore \quad F_s = F_c\cos\phi - F_t\sin\phi$

$= 1000\cos 28.63° - 400.85\sin 28.63° = 685.66$ N

39. In a grinding operation of a metal, specific energy consumption is 15J/mm^3. If a grinding wheel with a diameter of 200 mm is rotating at 3000 rpm to obtain a material removal rate of 6000 mm^3/min, then the tangential force on the wheel is ______________ N (round off to two decimal places).

Ans. 45.00 – 50.00

Exp:

E_{sp} = 15 J/mm^3

MRR = 6000 mm^3/min

N = 3000 rpm

D = 200 mm

$$E_{sp} = \frac{FcV}{LdV}$$

$$15 = \frac{F \times \frac{m}{\min}}{600 \frac{mm^3}{\min}}$$

$$V = \frac{\pi DN}{1m} = \frac{\pi(200)(3m)}{1000} = 600 \times 3.14$$

$$15 = \frac{F \times 600 \times 3\pi y}{6000}$$

F = 47.746 N ≃ 47.75 N

40. A 200 mm wide plate having a thickness of 20 mm is fed through a rolling mill with two rolls. The radius of each roll is 300 mm. The plate thickness is to be reduced to 18 mm in one pass using a roll speed of 50 rpm. The strength coefficient (K) of the work material flow curve is 300 MPa and the strain hardening exponent, n is 0.2. The coefficient of friction between the rolls and the plate is 0.1. If the friction is sufficient to permit the rolling operation then the roll force will be _____________kN (round off to the nearest integer).

Ans. 775 – 784

Exp:

$$h_{avg} = \frac{h_o + h_f}{2} = \frac{20+18}{2} = 19\,\text{mm}$$

$$L = \sqrt{R\Delta h} = \sqrt{300(20-18)} = 24.49\,\text{mm}$$

$$\Delta = \frac{h_{avg}}{L} = \frac{19}{24.49} = 0.77 < 1$$

Therefore, friction is important.

Now, $\varepsilon_f = \ln\left(\frac{h_f}{h_o}\right) = \ln\left(\frac{18}{20}\right) = -0.1054 = 0.01054$ (compression)

$$\bar{\sigma}_o = \frac{K\varepsilon_f^n}{1+n} = \frac{300\times(0.1054)^{0.1054}}{1+0.2} = 159.4 \text{ MPa (compression)}$$

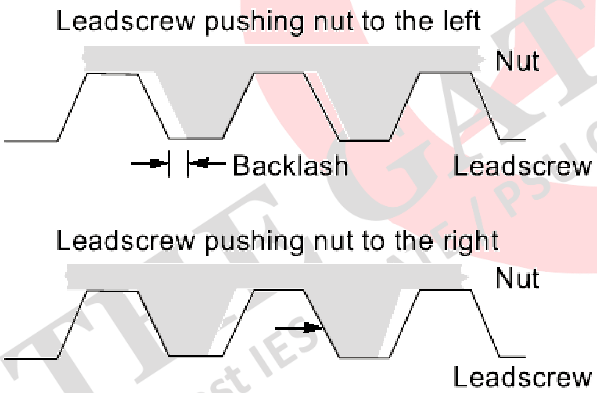
$$F = 1.15\sqrt{R\Delta h}\times b\times\bar{\sigma}_o\times\left(1+\frac{\mu\sqrt{R\Delta h}}{h_o+h_f}\right)$$

$$= 1.15\sqrt{300\times 2}\times 200\times 159.4\times\left(1+\frac{0.1\times\sqrt{300\times 2}}{20+18}\right) = 955.9 \text{ kN}$$

41. The XY table of a NC machine tool is to move from P(1,1) to Q(51,1); all coordinates are in mm. The pitch of the NC drive leadscrew is 1 mm. If the backlash between the leadscrew and the nut is 1.8^o, then the total backlash of the table on moving from P to Q is ___________ mm (round off to two decimal places).

Ans. 0.20 – 0.30

Exp:



P(1, 1) to P(51, 1) is 50 mm distance. As pitch of the leadscrew is is 1 mm. It has to move 50 rotation and in each rotation, backlash is 1.8°.Therefore, total backlash = 1.8 × 50°

In one rotation, i.e., 360° rotation – 1 mm movement take place.

$$\ln 1.8\times 50^o = \frac{1.8\times 50}{360}\text{mm} = 0.25 \text{ mm}$$

42. Consider a single machine workstation to which jobs arrive according to a Poisson distribution with a mean arrival rate of 12 jobs/hour. The process time of the workstation is exponentially distributed with a mean of 4 minutes. The expected number of jobs at the workstation at any given point of time is ___________ (round off to the nearest integer).

Ans. 4

Exp:

λ = 12 per hour, $\frac{1}{\mu}$ = 4 min/jobs, μ = 15 jobs/hr , $\rho = \frac{12}{15} = \frac{4}{5}$

$$L_s = \frac{\rho}{1-\rho} = \frac{\frac{4}{5}}{1-\frac{4}{5}} = 4 \text{ jobs}$$

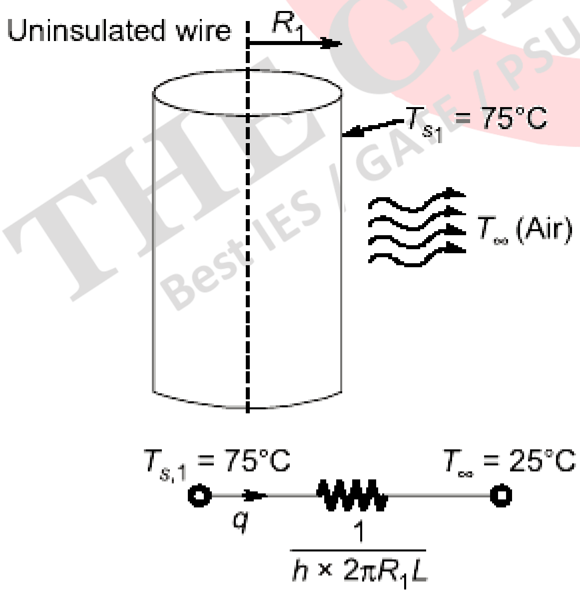
43. An uninsulated cylindrical wire of radius 1.0 mm produces electric heating at the rate of 5.0 W/m. The temperature of the surface of the wire is 75 °C when placed in air at 25 °C. When the wire is coated with PVC of thickness 1.0 mm, the temperature of the surface of the wire reduces to 55 °C. Assume that the heat generation rate from the wire and the convective heat transfer coefficient are same for both uninsulated wire and the coated wire. The thermal conductivity of PVC is ______________ W/m.K (round off to two decimal places).

Ans. 0.10 – 0.12

Exp:

For uninsulated wire:

Given, T_∞ = 25°C, R_1 = 1 mm, $\dot{q}_{gen}$ = 5 W/m, T_{s1} = 7.5°C

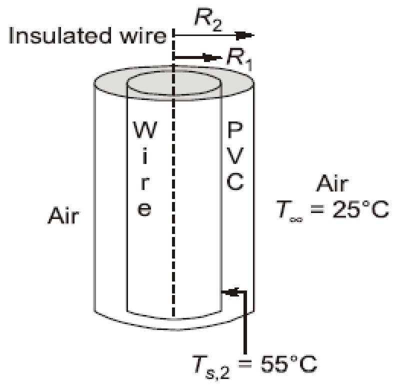


$$q = \dot{q}_{gen} \times L = \frac{T_{s1} - T_\infty}{\frac{1}{h \times 2\pi R_1 L}}$$

5 × L = h × (2π × 0.001)L × (75 – 25)

$h = 15.91\ W/m^2K$

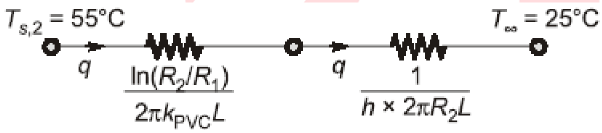
For insulated wire:



$R_2 = 2$ mm

Heat transfer rate after insulation kept on wire,

$q = q_{gen,\ wire} \times L = 5 \times L$



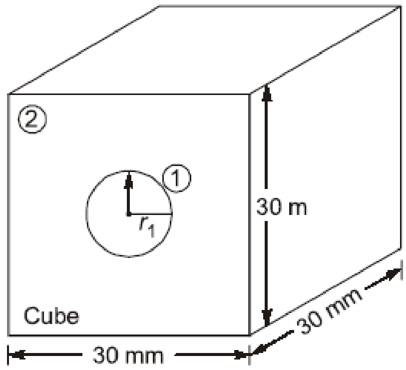
$$q = 5 \times L = \frac{55 - 25}{\dfrac{\ln(2/1)}{2\pi k_{PVC} L} + \dfrac{1}{15.91 \times 2\pi \times 0.002 \times L}}$$

$k_{PVC} = 0.11$ W/mK

44. A solid sphere of radius 10 mm is placed at the centroid of a hollow cubical enclosure of side length 30 mm. The outer surface of the sphere is denoted by 1 and the inner surface of the cube is denoted by 2. The view factor F_{22} for radiation heat transfer is ___________ (rounded off to two decimal places).

Ans. 0.76 – 0.78

Exp:



r_1 = 10 mm

$A_1 = 4\pi r_1^2$

$A_2 = 6 \times (30^2)$

$F_{12} = 1$

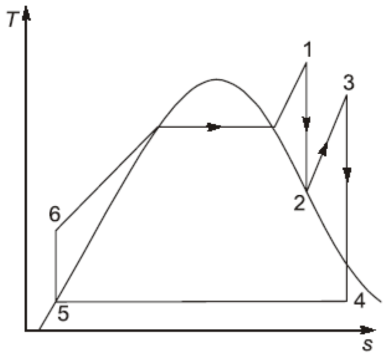
$$A_1F_{12} = A_2F_{21} \quad \Rightarrow \quad F_{21} = \frac{A_1}{A_2} = \frac{4\pi \times (10)^2}{6 \times (30)^2} = 0.2327$$

$F_{22} = 1 - F_{21} \simeq 0.77$

45. Consider a steam power plant operating on an ideal reheat Rankine cycle. The work input to the pump is 20 kJ/kg. The work output from the high pressure turbine is 750 kJ/kg. The work output from the low pressure turbine is 1500 kJ/kg. The thermal efficiency of the cycle is 50 %. The enthalpy of saturated liquid and saturated vapour at condenser pressure are 200 kJ/kg and 2600 kJ/kg, respectively. The quality of steam at the exit of the low pressure turbine is _______________ % (round off to the nearest integer).

Ans. 92 – 96

Exp:



$h_f = 200$ kJ/kg, $h_g = 2600$ kJ/kg, $w_p = 20$ kJ/kg $= h_6 - h_5$

$h_1 - h_2 = 750$ kJ/kg

$h_3 - h_4 = 1500$ kJ/kg

$$\eta = 0.5 = \frac{W_{NET}}{Q_s} = \frac{W_T - W_P}{Q_s}$$

$$0.5 = \frac{750 + 1500 - 20}{Q_s}$$

$Q_s = 4460$ kJ/kg

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$Q_R = 2230$ kJ/kg

$Q_R = h_4 - h_5$

$2230 = h_4 - 200$

$\Rightarrow \quad h_4 = 2430$ kJ/kg

$h_4 = h_f + x(h_g - h_f)$

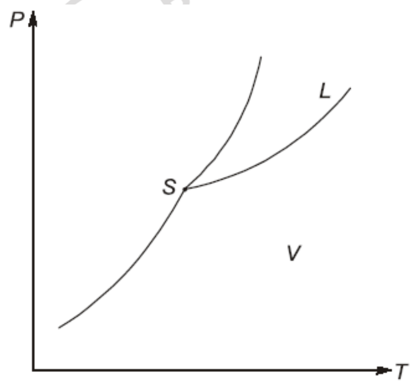
$2430 = 200 + x(2600 - 200)$

$x = 93\%$

46. In the vicinity of the triple point, the equation of liquid-vapour boundary in the P − T phase diagram for ammonia is lnP = 24.38 − 3063/T, where P is pressure (in Pa) and T is temperature (in K). Similarly, the solid-vapour boundary is given by lnP = 27.92 − 3754/T. The temperature at the triple point is ___________ K (round off to one decimal place).

Ans. 195.1 – 195.3

Exp:



Liquid vapour, $\ln P = 24.38 - \dfrac{3063}{T}$

Solid vapour, $\ln P = 27.92 - \frac{3754}{T}$

At triple point temperature of solid, liquid and vapour is same.

∴ Equating the above equations, we get

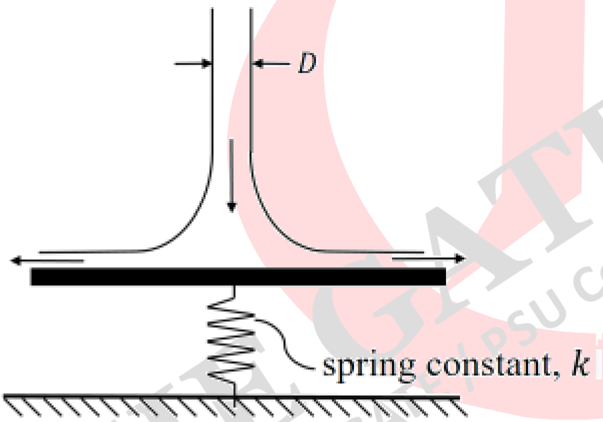
$$24.38 - \frac{3063}{T} = 27.92 - \frac{3754}{T}$$

Multiplying by T on both sides,

24.38 T – 3063 = 27.92 T – 3754

T = 195.19 ≃ 195.2 K

47. A cylindrical jet of water (density = 1000 kg/m^3) impinges at the center of a flat, circular plate and spreads radially outwards, as shown in the figure. The plate is resting on a linear spring with a spring constant k = 1 kN/m. The incoming jet diameter is D = 1 cm.



If the spring shows a steady deflection of 1 cm upon impingement of jet, then the velocity of the incoming jet is ____________ m/s (round off to one decimal place).

Ans. 11.2 – 11.4

Exp:

D = 1 cm,

δ = 1 cm

ρ = 1000 kg/m^3

K = 1 kN-m

Force due to jet = Spring force

$\rho A V^2 = kx$

$$10^3 \times \frac{\pi}{4} \times (0.01)^2 \times V^2 = 1 \times 10^3 \times 0.01$$

V = 11.28 m/s ≃ 11.3 m/s

48. A single jet Pelton wheel operates at 300 rpm. The mean diameter of the wheel is 2 m. Operating head and dimensions of jet are such that water comes out of the jet with a velocity of 40 m/s and flow rate of 5 m^3 /s. The jet is deflected by the bucket at an angle of 165°. Neglecting all losses, the power developed by the Pelton wheel is _______________ MW (round off to two decimal places).

Ans. 2.50 – 2.80

Exp:

P = ?

As we know that,

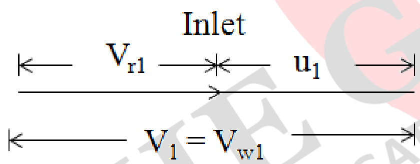
$P = \dot{m}\,[V_{w1} - V_{w2}]u$

N = 300 rpm

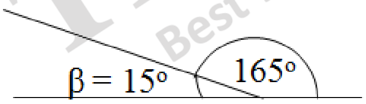
D = 2 m

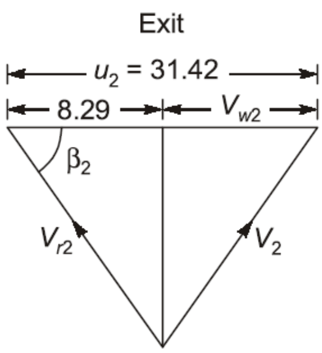
V_1 = 40 m/s

Q = 5 m^3/s



β = 180 – 165 = 15°





$$u = \frac{\pi DN}{60} = \frac{\pi \times 2 \times 300}{60}$$

$u_1 = u_2 = u = 31.42$ m/s

$V_{r1} = v_1 - u$

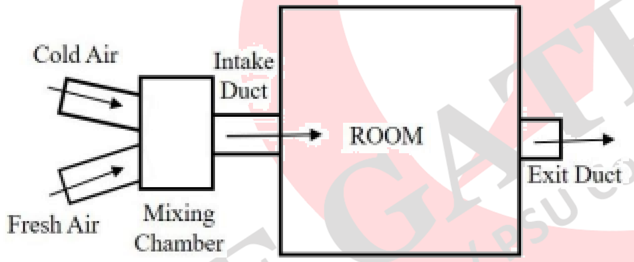
$V_{r1} = 40 - 31.42 = 8.58$

$V_{r1} = V_{r2} = 8.58$ m/s [neglecting blade friction]

$V_{r2}\cos\beta_2 = u_2 - V_{w2}$

$P = \rho Q[V_{w1} - V_{w2}] = 2.65$ MW

49. An air-conditioning system provides a continuous flow of air to a room using an intake duct and an exit duct, as shown in the figure. To maintain the quality of the indoor air, the intake duct supplies a mixture of fresh air with a cold air stream. The two streams are mixed in an insulated mixing chamber located upstream of the intake duct. Cold air enters the mixing chamber at 5 °C, 10^5 kPa with a volume flow rate of 1.25 m^3/s during steady state operation. Fresh air enters the mixing chamber at 34 °C and 10^5 kPa. The mass flow rate of the fresh air is 1.6 times of the cold air stream. Air leaves the room through the exit duct at 24 °C.



Assuming the air behaves as an ideal gas with $c_p = 1.005$ kJ/kg·K and R = 0.287 kJ/kg·K, the rate of heat gain by the air from the room is ____________ kW(round off to two decimal places).

Ans. 4.90 – 5.10

Exp:

1 : Cold air

2 : Hot air

$P_1V_1 = \dot{m}_1RT_1$

$105 \times 1.25 = \dot{m}_1 \times 0.287 \times 278$

$\dot{m}_1 = 1.645$ kg/sec

$\dot{m}_2 = 1.6 \times 1.645 = 2.632$ kg/sec

$\dot{m}_3 = 4.277$ kg/sec

After mixing : 0

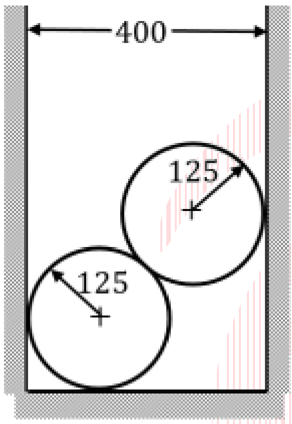
$\dot{m}_1 t_1 + \dot{m}_2 t_2 = \dot{m}_3 t_3$

$1.645 \times 5 + 2.632 \times 34 = 4.277\ t_3$

$t_3 = 22.85\ °C$

Heat gain = $h_4 - h_3 = \dot{m}_3 c_p(t_4 - t_3)$ = 4.277 2a2f 1.005 (24 – 22.85) = 4.96 kW

50. Two smooth identical spheres each of radius 125 mm and weight 100 N rest in a horizontal channel having vertical walls. The distance between vertical walls of the channel is 400 mm.

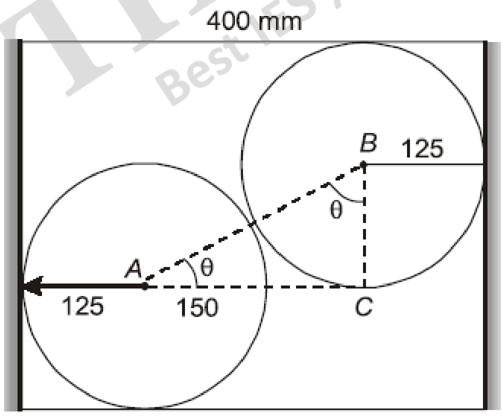


All dimensions are in mm

The reaction at the point of contact between two spheres is _________ N (round off to one decimal place).

Ans. 124.0 – 126.0

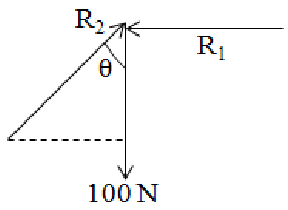
Exp:



$BC = 250^2 - 150^2$

$$\cos\theta = \frac{200}{250}$$

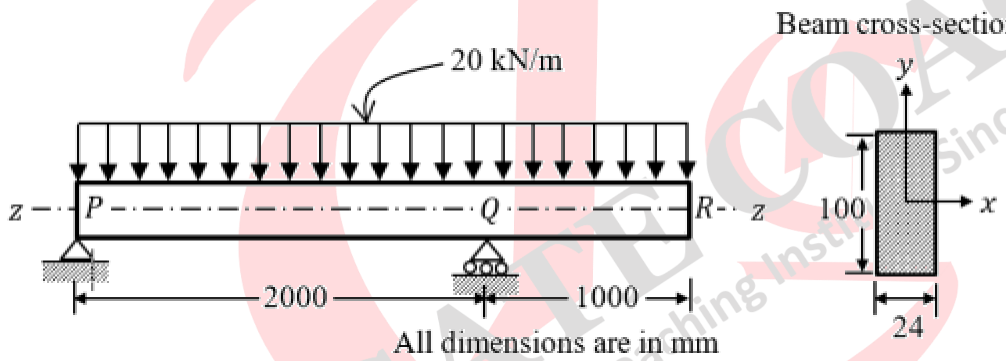
θ = 36.869°



$R_2 \cos\theta = 100$

$$R_2 = \frac{100 \times 250}{200} = 125.0 \text{ N}$$

51. An overhanging beam PQR is subjected to uniformly distributed load 20 kN/m as shown in the figure.



The maximum bending stress developed in the beam is ___________ MPa (round off to one decimal place).

Ans. 249.0 – 251.0

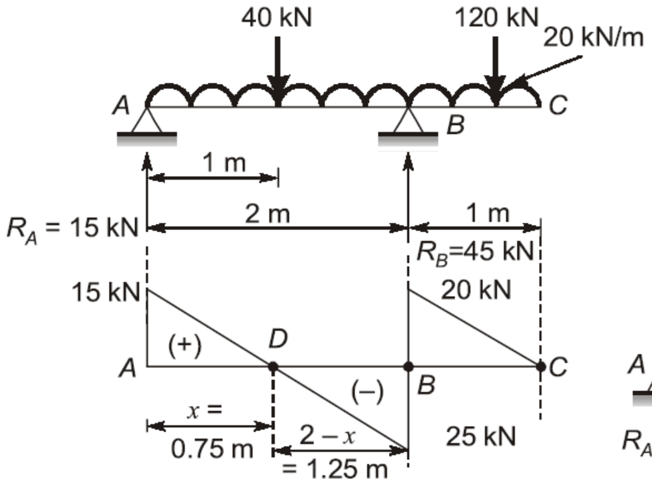
Exp:

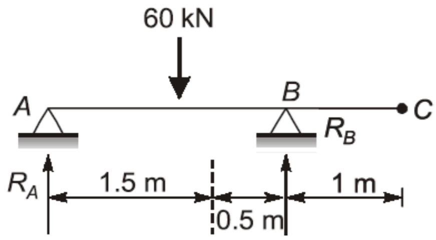
$$M_D - M_A = \frac{1}{2} \times 15 \times 0.75$$

M_D = 5.625 kN-m(s)

$\Sigma M_A = 60 \times 1.5 - R_B \times 2 = 0$

R_B = 45 kN (↑) ; R_A = 15 kN (↑)





Location of D:

$$\frac{15}{x} = \frac{25}{2-x}$$

$$x = \frac{3}{4}\text{ m}$$

$$M_C - M_B = \frac{1}{2} \times 20 \times 1$$

M_B = –10 kN-m

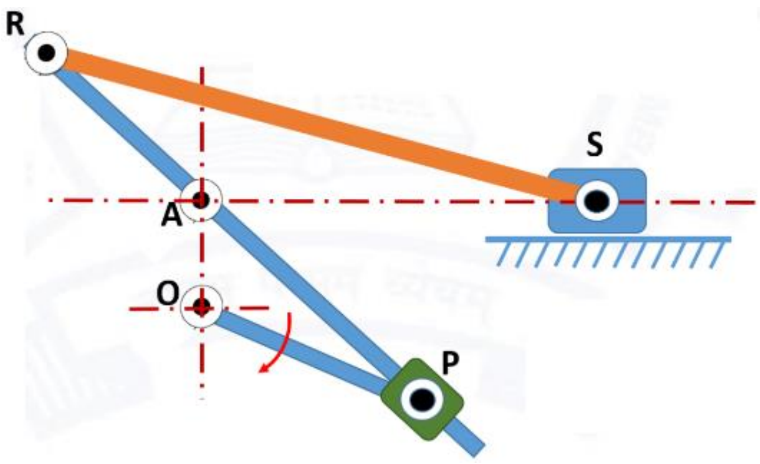
M_B = 10 kN-m (+1)

Max. B.M. = Larger of (M_B and M_D)

M_B = 10 kN-m (+1)

$$(\sigma_b)_{\max} = \frac{M_{\max}}{Z_{N,A}} = \frac{6 \times 10 \times 10^6}{24 \times 100^2} = 250 \text{ MPa}$$

52. The Whitworth quick return mechanism is shown in the figure with link lengths as follows: OP = 300 mm, OA = 150 mm, AR = 160 mm, RS = 450 mm.



The quick return ratio for the mechanism is _________ (round off to one decimal place).

Ans. 1.9 – 2.1

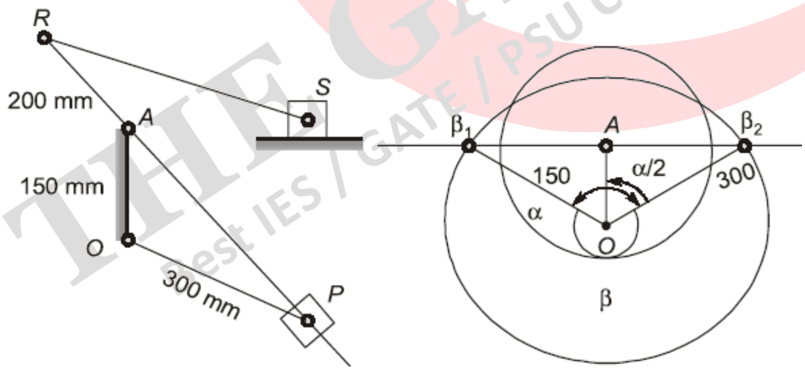
Exp:

$$\cos\frac{\alpha}{2} = \frac{150}{300} = \frac{1}{2}$$

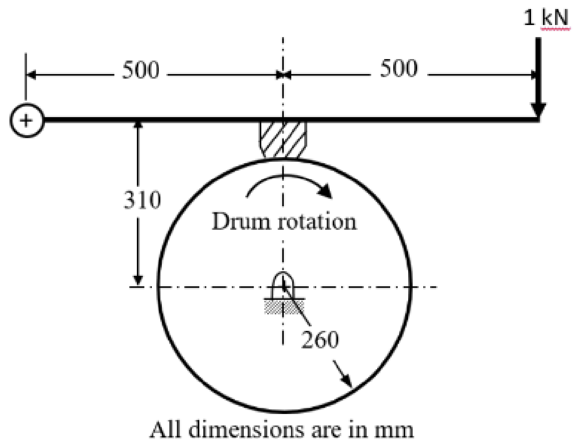
α = 120°

β = (360° – 120°) = 240°

$$QRR = \left(\frac{\beta}{\alpha}\right) = \frac{240}{120} = 2$$



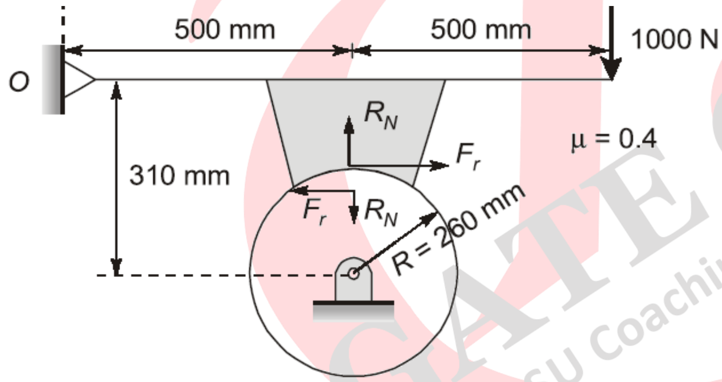
53. A short shoe drum (radius 260 mm) brake is shown in the figure. A force of 1 kN is applied to the lever. The coefficient of friction is 0.4.



The magnitude of the torque applied by the brake is ____________ N·m (round off to one decimal place).

Ans. 199.0 – 201.0

Exp:



Taking moment about 'O'

R_N (500) + F_Y [310 – 260] – 1000 × 1000 = 0

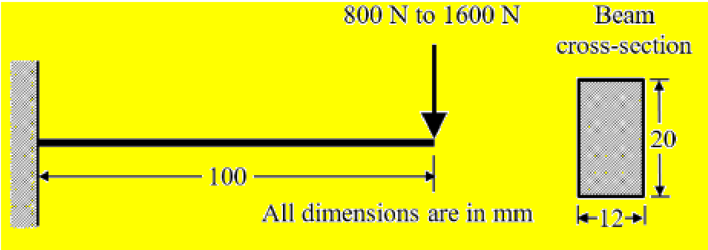
R_N (500) + 0.4 (R_N) (50) – 1000 × 1000 = 0

R_N = 1923.076 N

$F_r = \mu R_N$ = 769.23 N

$T_f = F_r \times R$ = 199.9 N-m

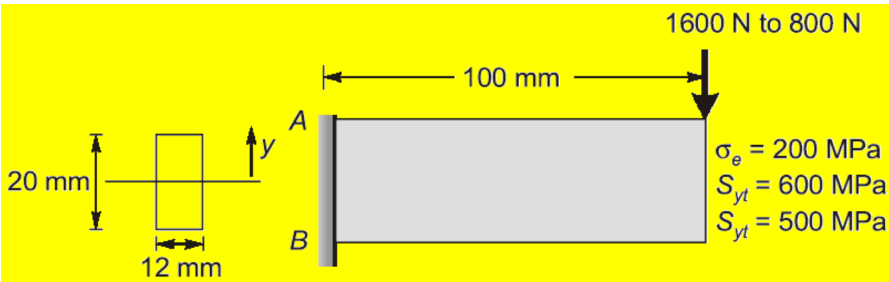
54. A machine part in the form of cantilever beam is subjected to fluctuating load as shown in the figure. The load varies from 800 N to 1600 N. The modified endurance, yield and ultimate strengths of the material are 200 MPa, 500 MPa and 600 MPa, respectively.



The factor of safety of the beam using modified Goodman criterion is __________ (round off to one decimal place).

Ans. 1.9 – 2.1

Exp:



A : Critical Point

$$\sigma_{\max,A} = \sigma_{b,\max} \text{ at A due to 1600 N} = \frac{6M}{bd^2} = \frac{6\times1600\times200}{12\times(2\sigma)^2}$$

σ_{max} = 200 MPa

$$\sigma_{\min,A} = \sigma_{b,\max} \text{ at A due to 800 N} = \frac{6\times800\times200}{12\times(20)^2} = 100 \text{ MPa}$$

$$\text{Modified Goodman} = \frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{\sigma_e} \le \frac{1}{N}$$

$$\sigma_m = \left|\frac{\sigma_{\max}+\sigma_{\min}}{2}\right| = 150 \text{ MPa}$$

$$\sigma_a = \left|\frac{\sigma_{\max}-\sigma_{\min}}{2}\right| = 50 \text{ MPa}$$

$$\frac{150}{600} + \frac{50}{200} \le \frac{1}{N}$$

N = 2

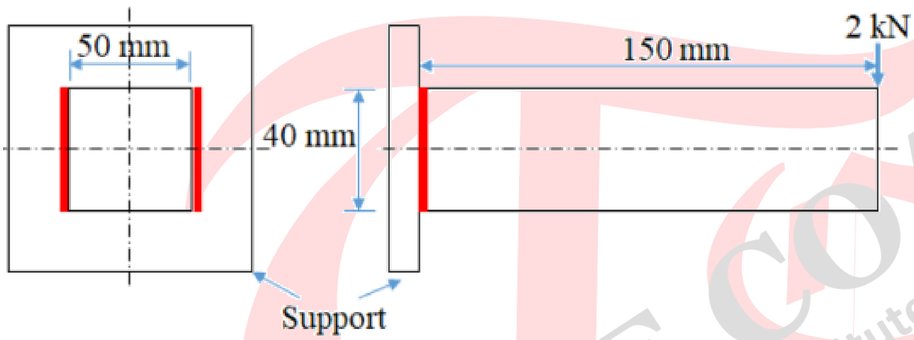
Langer, $\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} \le \frac{1}{N}$

$\frac{100}{500} + \frac{50}{500} \le \frac{1}{N}$ (N ≤ 2.5)

N = 2.5

Modified Goodman = Safe result of [Goodman or Langer].

55. A cantilever beam of rectangular cross-section is welded to a support by means of two fillet welds as shown in figure. A vertical load of 2 kN acts at free end of the beam.



Considering that the allowable shear stress in weld is 60 N/mm², the minimum size (leg) of the weld required is __________ mm (round off to one decimal place).

Ans. 6.4 – 6.9

Exp:

$$\tau_{\max} = \frac{2 \times 10^3}{0.707t(40) \times 2} = \frac{35.36}{t} MPa$$

$$\sigma_{\max} = \frac{M_{\max}}{I_{NA}} \cdot \tau_{\max} = \frac{2000 \times 150 \times 20}{\frac{0.707t(40)^3 \times 2}{12}}$$

$$\sigma_{\max} = \frac{795.615}{t} MPa$$

MSST, $\sqrt{\sigma_{\max}^2 + 4\tau^2} \le 2\left(\frac{S_{ys}}{N}\right)$

$$\sqrt{\left(\frac{795.615}{t}\right)^2 + 4\left(\frac{35.36}{t}\right)^2} \le 2 \times 60$$

t ≃ 6.7 mm