GATE – 2021

Instrumentation Engineering (IN)

General Aptitude (GA)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

1. Getting to the top is ________ than staying on top.

(a) more easy

(b) much easy

(c) easiest

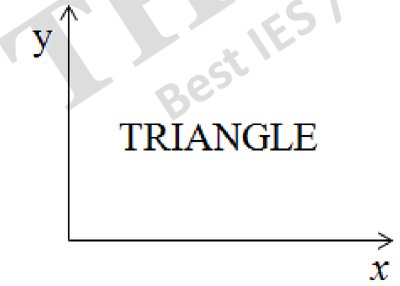
(d) easier

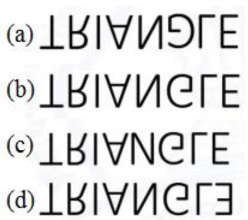
Ans. d

Exp:

When the comparison is between the two things we use the second degree of the adjective. The degree form of easy are : easy → easier → easiest.

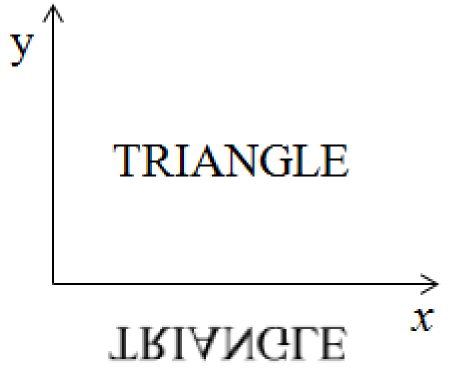
2. The mirror image on the above text about the x-axis is





Ans. b

Exp:



3. In a company, 35% of the employees drink coffee, 40% of the employees drink tea and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?

(a) 15

(b) 25

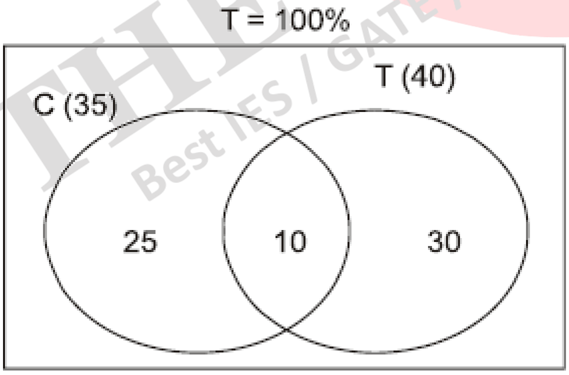
(c) 35

(d) 40

Ans. c

Exp:

Percent of employees drink neither tea nor coffee = 100 – 25 – 10 – 30 = 35



4. ⊕ and ⊙ are two operators on numbers p and q such that

$$p \oplus q = \frac{p^2+q^2}{pq} \qquad \text{and} \qquad p \odot q = \frac{p^2}{q};$$

If $x \oplus y = 2 \odot 2$, then $x =$

(a) $\frac{y}{2}$

(b) y

(c) $\frac{3y}{2}$

(d) 2y

Ans. b

Exp:

Given that, If $x \oplus y = 2 \odot 2$,

$$\frac{x^2 + y^2}{xy} = \frac{2^2}{2}$$

$$x^2 + y^2 = 2xy$$

$$(x - y)^2 = 0$$

$x = y$

5. Four persons P, Q, R and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. P and R cannot sit adjacent to each other. S should be seated to the right of Q. The number of distinct seating arrangements possible is :

(a) 2

(b) 4

(c) 6

(d) 8

Ans. c

Exp:

Following cases can be

PQRS, PQSR, QPSR, RQSP, QRSP, RQPS

Q. 6 – Q.10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: –2/3).

6. Statement: Either P marries Q or X marries Y

Among the option below, the logical NEGATION of the above statement is:

(a) P does not marry Q and X marries Y

(b) Neither P marries Q nor X marries Y

(c) X does not marry Y and P marries Q

(d) P marries Q and X marries Y

Ans. b

Exp:

The statement says only one of these two action will happen, its NEGATION should be a confirmed action.

7. Consider two rectangular sheets, Sheet M and Sheet N of dimensions 6 cm × 4 cm each.

Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.

Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.

Folding operation 1 is carried out on Sheet M three times.

Folding operation 2 is carried out on Sheet N three times.

The ratio of perimeter of the final folded shape of Sheet N to the final folded shape of Sheet M is ________

(a) 13 : 7

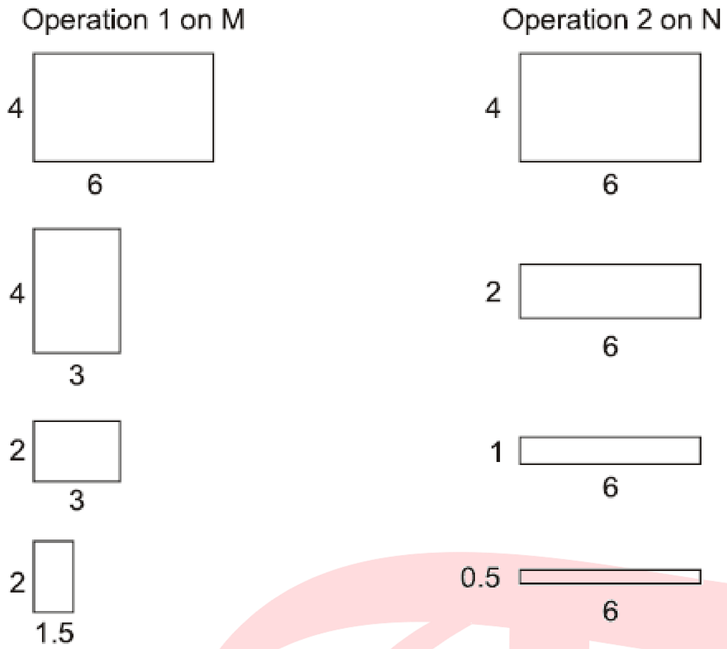
(b) 3 : 2

(c) 7 : 5

(d) 5 : 13

Ans: a

Exp:

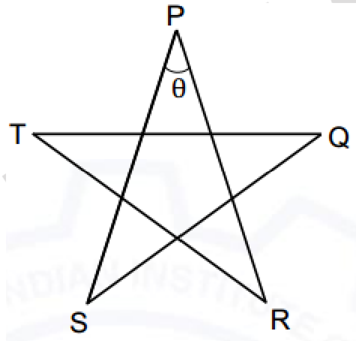


$(\text{Perimeter})_M = 2(2 + 1.5) = 7$ $(\text{Perimeter})_N = 2(0.5 + 6) = 13$

Required ratio $= \dfrac{13}{7}$

8. Five line segments of equal lengths, PR, PS, QS, QT and RT are used to form a star as shown in the figure below.

The value of θ is ________(degree).



(a) 36

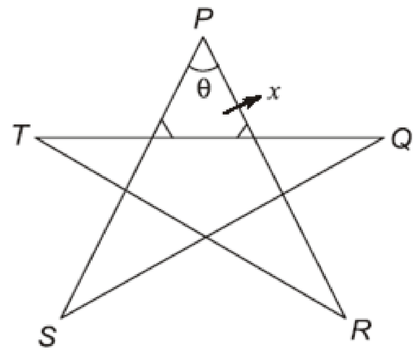
(b) 45

(c) 72

(d) 108

Ans. a

Exp:



Sum of angle formed at the pentagon = 540°

Each angle of pentagon = $\frac{540}{5} = 108^o$

$\angle x$ = 180 – 108 = 72°

Sum of angle of triangle = 180°

72° + 72° + θ = 180°

θ = 36°

9. A function, λ, is defined by

$$\lambda(p, q) = \begin{cases} (p-q)^2, & if\ p \geq q \\ p+q, & if\ p < q \end{cases}$$

The value of the expression $\frac{\lambda(-(-3+2),(-2+3))}{(-(2+1))}$ is

(a) –1

(b) 0

(c) $\frac{16}{3}$

(d) 16

Ans. b

Exp:

$$\frac{\lambda(-(-3+2),(-2+3))}{(-(2+1))} = \lambda\frac{(1,1)}{1} = \lambda(1,1)$$

So, 1st definition will be applicable as p = q.

Hence, $\lambda(1, 1) = (1 - 1)^2 = 0$

10. Human have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So, far as we know, no other species possesses the ability. This skill is so important that we have different worlds to refer to its different flavours, such as imagination invention and innovation.

Based on the above passage, which one of the following is TRUE?

(a) No spices possess the ability to construct worlds in their minds.

(b) The terms imagination, invention and innovation refer to unrelated skill.

(c) We do not know of any species other than humans who possess the ability to construct mental worlds.

(d) Imagination, invention and innovation are unrelated to the ability to construct mental worlds.

Ans. c

Instrumentation Engineering (IN)

Q.1 – Q.8 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

1. Consider the row vectors v = (1, 0) and w = (2, 0). The rank of the matrix $M = 2v^Tv + 3w^Tw$, where the superscript T denotes the transpose, is

(a) 1

(b) 2

(c) 3

(d) 4

Ans. a

Exp:

$$A = 2\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} + 3\begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}2 & 0\end{bmatrix}$$

$$= 2\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + 3\begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}14 & 0\\0 & 0\end{bmatrix}$$

$\rho(A) = 1$

2. Consider the sequence $x_n = 0.5x_{n-1} + 1$, n = 1, 2, ... with $x_0 = 0$. Then $\lim_{n\to\infty} x_n$ is

(a) 0

(b) 1

(c) 2

(d) ∞

Ans. c

Exp:

From the given sequence, we can find the value like,

$$x_n = \frac{x_{n-1}}{2} + 1 \quad \Rightarrow \quad x_1 = 1, x_2 = \frac{3}{2}, x_3 = \frac{7}{4}, x_4 = \frac{15}{8}, x_5 = \frac{31}{16}$$

Now, sequence becomes,

$$1, \frac{3}{2}, \frac{7}{2^2}, \frac{15}{2^3}, \frac{31}{2^4}, \ldots \frac{2^n - 1}{2^{n-1}}, \ldots$$

i.e., $x_n = \dfrac{2^n - 1}{2^{n-1}} = \dfrac{2^n - 1}{2^n/2} = 2\left[1 - \dfrac{1}{2^n}\right]$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} 2\left(1 - \frac{1}{2^n}\right) = 2\left(1 - \frac{1}{\infty}\right) = 2$$

3. An infinitely long line, with uniform positive charge density, lies along the z- axis. In cylindrical coordinates (r,∅, z), at any point $\vec{P}$ not on the z-axis, the direction of the electric field is

(a) $\hat{r}$

(b) $\hat{\varnothing}$

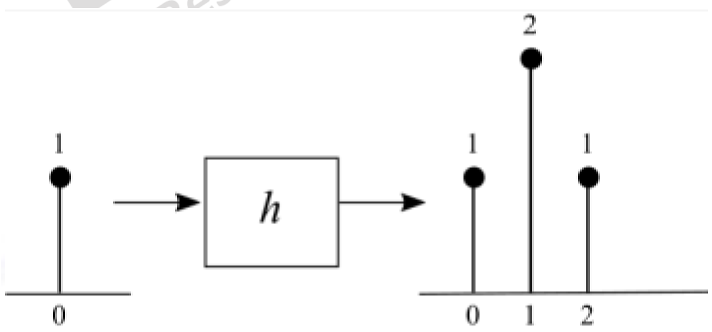
(c) $\hat{z}$

(d) $\dfrac{(\hat{r} + \hat{z})}{\sqrt{2}}$

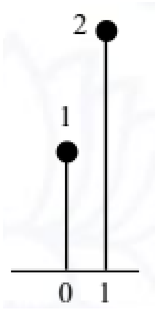
Ans. a

Exp:

4. The input-output relationship of an LTI system is given below:



For an input x[n] shown below



the peak value of the output when x[n] passes through h is ____.

(a) 2

(b) 4

(c) 5

(d) 6

Ans. c

Exp:

When $x[n] = \delta[n]$, then $y[n] = h[n]$

$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

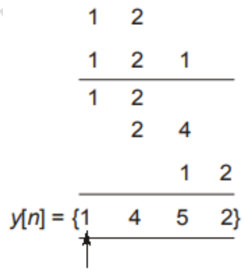
$= \{1, 2, 1\}$

Given, $x'[n] = \delta[n] + 2\delta[n-1]$

$x'[n] = \{1, 2\}$

$y[n] = x'[n] * h[n]$

Use Sum by column method:



Peak value at the output is (5).

5. In an ac main, the rms voltage V_{ac}, rms current I_{ac} and power W_{ac} are measured as: V_{ac} = 100 V ± 1%, I_{ac} = 1 A ± 1% and W_{ac} = 50 W ± 2% (errors are with respect to readings). The percentage error in calculating the power factor using these readings is

(a) 1%

(b) 2%

(c) 3%

(d) 4%

Ans. d

Exp:

V_{aC} = 100 ± 1%, I_{aC} = ±1%, W_{aC} = 50W ± 2%

P = VIcos(ϕ) ⇒ pf = cos(ϕ) = $\frac{P}{VI} = \frac{50}{100 \times 1} = 0.5$

% Error = $\pm\left[\frac{\delta P}{P} + \frac{\delta V}{V} + \frac{\delta I}{I}\right]$ = ± [2% + 1% + 1%] = ± 4%

6. Let u(t) denote the unit step function. The bilateral Laplace transform of the function

f(t) = $e^t u(-t)$ is ____.

(a) $\frac{1}{s-1}$ with real part of s < 1

(b) $\frac{1}{s-1}$ with real part of s > 1

(c) $\frac{-1}{s-1}$ with real part of s < 1

(d) $\frac{-1}{s-1}$ with real part of s > 1

Ans. c

Exp:

f(t) = $e^t u(-t)$

As we know, $e^{-t}u(t) \leftrightarrow \frac{1}{s+1}, \sigma > -1$

By using time-reversal property of Laplace transform,

$e^{-t}u(t) \leftrightarrow \frac{1}{-s+1}, -\sigma > -1$

$\therefore \quad e^{t}u(-t) \quad \longleftrightarrow \quad \frac{-1}{s-1}, \sigma < 1$

7. Input-output characteristic of a temperature sensor is exponential for a

(a) Thermistor

(b) Thermocouple

(c) Resistive Temperature Device (RTD)

(d) Mercury thermometer

Ans. a

8. The signal sin($\sqrt{2\pi t}$) is

(a) periodic with period T = $\sqrt{2\pi}$

(b) not periodic

(c) periodic with period T = 2π

(d) periodic with period T = $4\pi^2$

Ans. b

Exp:

$$x(t) = \sin\left(\sqrt{2\pi t}\right)$$

• For negative values of ‘t’, sine-function will apply on imaginary values.

• For positive values of ‘t’, sine-function will apply on real-values.

• So, there will not be repetition in the waveform.

$\therefore$ We can draw waveform only in the RHS.

But we cannot draw waveform in the LHS.

• Therefore, sin $\sqrt{2\pi t}$ is non-periodic.

Q.9 – Q.11 Multiple Select Question (MSQ), carry ONE mark each (no negative marks).

9. The step response of a circuit is seen to have an oscillatory behaviour at the output with oscillations dying down after some time. The correct inference(s) regarding the transfer function from input to output is/are

(a) that it is of at least second order.

(b) that it has at least one pole-pair that is underdamped.

(c) that it does not have a real pole.

(d) that it is a first order system.

Ans. a, b

Exp:

If system has oscillatory behaviour it should be under damped.

For underdamped system, least order of system should be 2.

For 3^{rd} order system, it can have a real system with under damped in nature. End of Solution

10. For a 4-bit Flash type Analog to Digital Convertor (ADC) with full scale input voltage range "V", which of the following statement(s) is/are true?

(a) The ADC requires 15 comparators.

(b) The ADC requires one 4 to 2 priority encoder and 4 comparators.

(c) A change in the input voltage by $\frac{V}{16}$ will always flip MSB of the output.

(d) A change in the input voltage by $\frac{V}{16}$ will always flip the LSB of the output.

Ans. a, d

Exp:

Given, 4 bit flash ADC, n = 4

Resolution = $\frac{V_R}{2^n} = \frac{V_R}{2^4} = \frac{V_R}{2^{16}}$

Any change $\frac{V_R}{16}$ changes LSB.

Example, Input : 0 V → 0000

$\frac{V_R}{16} \rightarrow 0001$

No. of comparators required for 4-bit flash type ADC = $2^4 - 1 = 15$

11. A 16-bit microprocessor has twenty address lines (A0 to A19) and 16 data lines. The higher eight significant lines of the data bus of the processor are tied to the 8-data lines of a 16 Kbyte memory that can store one byte in each of its 16K address locations. The memory chip should map onto contiguous memory locations and occupy only 16 Kbyte of memory space. Which of the following statement(s) is/are correct with respect to the above design?

(a) If the 16 Kbyte of memory chip is mapped with a starting address of 80000H, then the ending address will be 83FFFH.

(b) The active high chip-select needed to map the 16 Kbyte memory with a starting address at F0000H is given by the logic expression (A19 · A18 · A17 · A16).

(c) The 16 Kbyte memory cannot be mapped with contiguous address locations with a starting address as 0F000H using only A19 to A14 for generating chip select.

(d) The above chip cannot be interfaced as the width of the data bus of the processor and the memory chip differs.

Ans. a, c

Exp:

The type of microprocessor → 16 bit

Data bus length → 16 bit

Number of address lines→ 20 ($A_{19} - A_0$)

Memory chip connected → 16 kB

Given condition is memory chip of 16 kB should be mapped onto contiguous memory locations. For a 16 kB memory chip, number of address lines required,

$2^n = N = 16 \text{ kB} = 2^{14}$

n = 14 i.e. $A_{13} - A_0$

Remaining 6 lines can be used for chip select i.e. $A_{19} - A_{14}$.

As per option 153, if starting address is 0F000H i.e.

$A_{19}\ A_{18}\ A_{17}\ A_{16}\ A_{15}\ A_{14}\ A_{13}\ \ldots\ldots\ldots\ldots\ A_0$

$\underbrace{0\ 0\ 0\ 0\ 1\ 1\ 1\ 1}_{\text{Chip select lines}}\ 0\ 0\ 0\ 0\ 0$

The ending for starting address of 0F000H is 12FFFH but the chip select lines get modified. So, the statement is correct.

If starting addressing 80000H than ending address would be 83FFFH which does not change A_{19} to B_{14} lines i.e. 100000.

For 16 kB to find ending address 3FFF H should be added to starting address.

Q.12 – Q.25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

12. A single-phase transformer has a magnetizing inductance of 250 mH and a core loss resistance of 300 Ω, referred to primary side. When excited with a 230 V, 50 Hz sinusoidal supply at the primary, the power factor of the input current drawn, with secondary on open circuit, is ____ (rounded off to two decimal places).

Ans. 0.24 – 0.26

Exp:

$L = 250 \times 10^{-3}$ H; $R_0 = 300\ \Omega$

$$B_L = \frac{1}{2\pi f\, L} = \frac{1}{2\pi(50)\,250\times10^{-3}} = 0.012732$$

$$G_o = \frac{1}{300} = 3.33\times10^{-3}$$

$$\cos\theta = \frac{G_o}{\sqrt{G_o^2 + B_L^2}} = 0.253 \text{ lag}$$

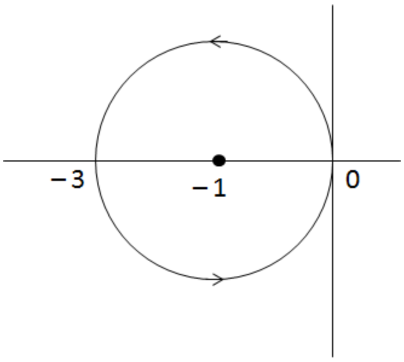
13. Taking N as positive for clockwise encirclement, otherwise negative, the number of encirclements N of (−1, 0) in the Nyquist plot of $G(s) = \frac{3}{s-1}$ is ____.

Ans. –1

Exp:

$$\text{OLTF} = \frac{3}{s-1}$$

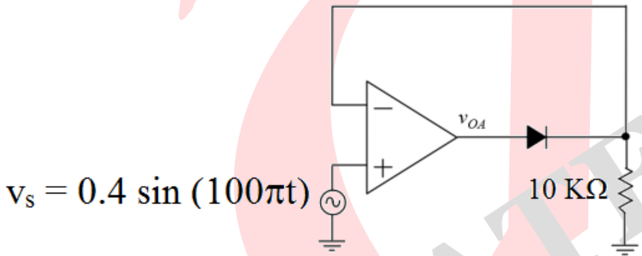
Nyquist plot



Since N is +ve in CW and –ve in ACW as in the question,

∴ $N = -1$

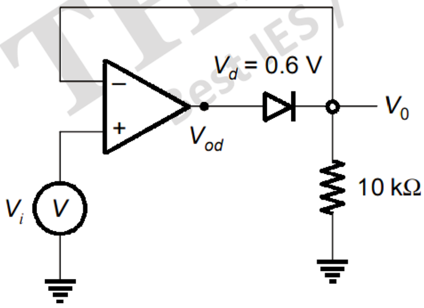
14. The diode used in the circuit has a fixed voltage drop of 0.6 V when forward biased. A signal v_s is given to the ideal Op-Amp as shown. When v_s is at its positive peak, the output (v_{OA}) of the Op-Amp in volts is ________.



Ans. 1

Exp:

Given circuit is called precision diode.



$V_i = 0.4 \sin \omega t$

Let diode be OFF.

Then $V^- = V_0 = 0$ and $V^+ = V$;

If $V^+ < V^-$ i.e., $V_i < 0$ then $V_{od} = -V_{sat}$ and diode remains OFF

$\Rightarrow \quad V_0 = 0$

If $V^+ > V^-$ i.e., $V_i > 0$ then V_{od} becomes +ve.

Now diode becomes ON and op-amp will be closed-loop.

$\Rightarrow V^- = V^+ \Rightarrow V_0 = V_i$

If V_i has peak value

i.e. $V^i = 0.4$ V

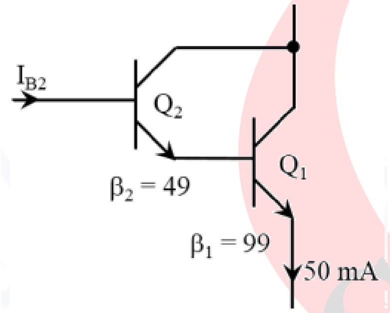
$\Rightarrow V_0 = 0.4$ V

Now $V_{od} - V_0 = V_d$

$\Rightarrow V_{od} = V_d + V_0 = 0.6 + 0.4$

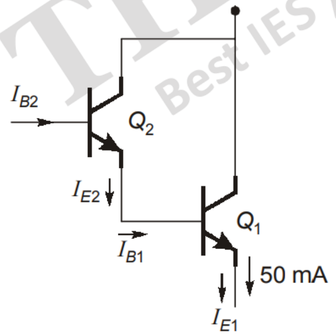
$\Rightarrow V_{od} = 1$ V

15. The transistor Q_1 has a current gain $\beta_1 = 99$ and the transistor Q_2 has a current gain $\beta_2 = 49$. The current I_{B2} in microampere is _______.



Ans. 10

Exp:



$\beta_1 = 99$, $\beta_2 = 49$

$I_{E1} = 50$ mA

$I_{E2} = I_{B1} = \dfrac{I_{E1}}{1+\beta_1}$

$$\Rightarrow \qquad I_{E2} = \frac{50}{100} = 500\,\mu A$$

$$I_{B2} = \frac{I_{E2}}{1+\beta_2} = \frac{500}{50} = 10\mu A$$

16. A 300 V, 5 A, LPF wattmeter has a full scale of 300 W. The wattmeter can be used for loads supplied by 300 V ac mains with a maximum power factor of ___________ (rounded off to one decimal place).

Ans. 0.2

Exp:

For low power factor wattmeter,

VI cosϕ = 300

300 × 5 cosϕ = 300

cosϕ = 0.2

17. A 10-bit ADC has a full-scale of 10.230 V, when the digital output is $(11\ 1111\ 1111)_2$. The quantization error of the ADC in millivolt is _________.

Ans. 4.9 – 5.1

Exp:

Given, 10 bit ADC, Full scale = 10.23 V

Resolution = $\frac{V_{FS}}{2^n - 1} = \frac{10.23}{2^{10} - 1}$ = 10 mV

Quantization error = $\frac{step\ size}{2}$ = 5 mV (Step size = resolution of ADC)

18. A strain gauge having nominal resistance of 1000 Ω has a gauge factor of 2.5. If the strain applied to the gage is 100 μm/m, its resistance in ohm will change to ____________ (rounded off to two decimal places).

Ans. 1000.25

Exp:

For strain gauge = $\frac{\delta R}{R} = G_f \times \in \quad (\in = \text{strain})$

$\delta R = 1000 \times 2.5 \times 100 \times 10^{-6} = 0.25\ \Omega$

Resistance of the strain gauge changes to $1000 + 0.25 = 1000.25\Omega$

19. Given: Density of mercury is 13,600 kg/m^3 and acceleration due to gravity is 9.81 m/s^2. Atmospheric pressure is 101 kPa. In a mercury U-tube manometer, the difference between the heights of the liquid in the U-tube is 1 cm. The differential pressure being measured in pascal is ______ (rounded off to the nearest integer).

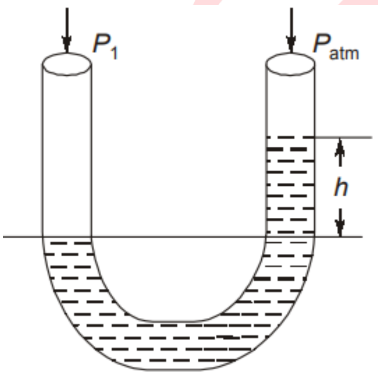
Ans. 1333 - 1360

Exp:

Density of monometric fluid (ρ) = 13,600 kg/m^3

Acceleration due to gravity is (g) = 9.81 m/s^2

Height indicated (h) = 1 cm $P1 = P_{atm} + \rho gh$



Differential pressure

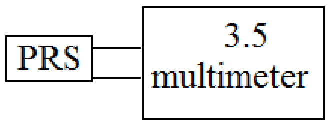
$\Delta P = P_1 - P_{atm} = \rho gh$

$\Delta P = 13600 \times 9.81 \times 10 \times 10^{-2} = 1334.16$ Pa

20. A piezoresistive pressure sensor has a sensitivity of 1 (mV/V)/kPa. The sensor is excited with a dc supply of 10 V and the output is read using a $3\frac{1}{2}$ digit 200 mV full-scale digital multimeter. The resolution of the measurement set-up, in pascal is ___.

Ans. 10

Exp:



Total counts = 2000 counts

Maximum voltage = 200 mV

$$Sensitivity = \frac{\frac{1mV}{V}}{1k\,Pascal}$$

Input voltage = 10 V

Full scale output, V_o = 200 mV

Also, $Sensitivity = \frac{\frac{200\,mV}{10V}}{P_{in}}$

From the above two equations,

P_{in} = 20 K Pascal

Resolution = $\frac{20\,K\ pascal}{2000\,counts} = 10\ pascal$

21. An amplitude modulation (AM) scheme uses tone modulation, with modulation index of 0.6. The power efficiency of the AM scheme is ____ % (rounded off to one decimal place).

Ans.15.0 – 15.5

Exp:

Amplitude modulation, μ = 0.6

Assuming sinusoidal message

Modulation efficiency, η = $\frac{\mu^2/2}{1+\frac{\mu^2}{2}} \times 100$

$$\%\eta = \frac{(0.6)^2}{2+(0.6)^2} = \frac{0.36}{2+0.36} \times 100 \approx 15.2\ \%$$

22. When the movable arm of a Michelson interferometer in vacuum (n = 1) is moved by 325 μm, the number of fringe crossings is 1000. The wavelength of the laser used in nanometers is ____.

Ans. 650

Exp:

Given, d = 325 mm

path difference, = 2d = 2 × 325 mm

path difference = nλ

∴ $2d = n\lambda$ ⟹ $n = \dfrac{2d}{\lambda} = \dfrac{2\times 325\,\mu m}{1000}$

∴ n = 650 nm

23. Consider the function $f(x) = -x^2 + 10x + 100$. The minimum value of the function in the interval [5, 10] is ___.

Ans. 100

Exp:

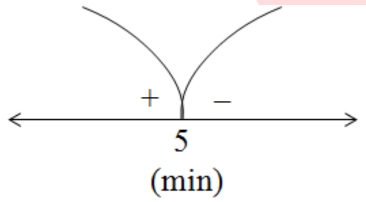
$f'(x) = -2x + 10$

$f'(5) = 0$

$f'(10) = -100$

∵ $f'(x) \leq 0$ in [5, 10] so f(x) is decreasing function.

Min $f(x) = f(10) = -(10)^2 + 10(10) + 100 = -100 + 100 + 100 = 100$



24. Let $f(z) = \dfrac{1}{z^2+6z+9}$ defined in the complex plane. The integral $\int_C f(z)$ over the contour of a circle c with center at the origin and unit radius is____.

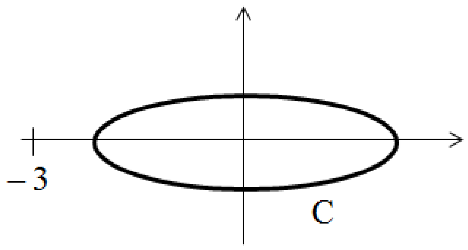
Ans. 0

Exp:

$f(z) = \dfrac{1}{z^2+6z+9} = \dfrac{1}{(z+3)^2}$

poles of f(z), $(z+3)^2 = 0$ ⟹ $z = -3$ (Double pole)

C : $|z| = 1$



$\because$ Pole lies outside 'C' so by CIT $\int_C f(z)\,dz = 0$

25. The determinant of the matrix M shown below is ____.

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Ans. 4

Exp:

Expanding along R_1

$$|A| = +(1)\begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 3 \\ 0 & 2 & 1 \end{vmatrix} - (2)\begin{vmatrix} 3 & 4 & 0 \\ 0 & 4 & 3 \\ 0 & 2 & 1 \end{vmatrix}$$

= (1)[4(4 – 6)] –2[3(4 –6)] = 4

Q.26 – Q.36 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: – 2/3).

26. $f(z) = (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + \cdots$ is the series expansion of

(a) $\dfrac{-1}{z(z-1)}$ for $|z-1| < 1$

(b) $\dfrac{1}{z(z-1)}$ for $|z-1| < 1$

(c) $\dfrac{1}{z(z-1)^2}$ for $|z-1| < 1$

(d) $\dfrac{-1}{z(z-1)}$ for $|z-1| < 1$

Ans. b

Exp:

$$f(z)=(z-1)^{-1}\left[1-\frac{1}{(z-1)^{-1}}+\frac{(z-1)}{(z-1)^{-1}}-\frac{(z-1)^{2}}{(z-1)^{-1}}+\ldots\right]$$

$$=(z-1)^{-1}\left[1-(z-1)+(z-1)^{2}-(z-1)^{3}+\ldots\right]$$

We know that, $(1+x)^{-1}=1-x+x^2-x^3+\ldots$; for $|x|<1$

$$=\frac{1}{(z-1)}\left[1+(z-1)\right]^{-1};\quad for\ |z-1|<1$$

$$=\frac{1}{(z-1)}\left[z^{-1}\right];\qquad ;\quad for\ |z-1|<1$$

$$=\frac{1}{z(z-1)};\quad for\ |z-1|<1$$

27. A single-phase transformer has maximum efficiency of 98 %. The core losses are 80 W and the equivalent winding resistance as seen from the primary side is 0.5 Ω. The rated current on the primary side is 25 A. The percentage of the rated input current at which the maximum efficiency occurs is

(a) 35.7%

(b) 50.6%

(c) 80.5%

(d) 100%

Ans. b

Exp:

At maximum efficiency,

Core loss = Copper losses = 80 W

Copper loss at rated current = $I^2R = (25)^2 \times 0.5 = 312.5$ W

When maximum efficiency occurred,

$$I_m^2 R = 80$$

$$I_m^2=\frac{80}{0.5}=160$$

I_m = 12.649 A

% of rate of current = $\frac{12.649}{25} \times 100$ = 50.60 %

28. A slip-ring induction motor is expected to be started by adding extra resistance in the rotor circuit. The benefit that is derived by adding extra resistance in the rotor circuit in comparison to the rotor being shorted is

(a) The starting torque would be higher.

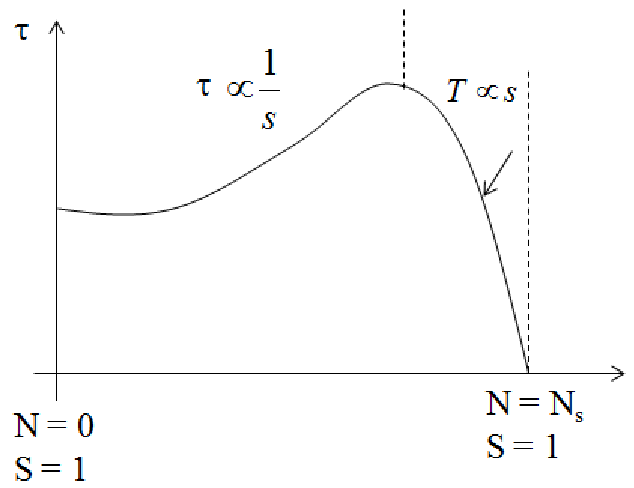
(b) The power factor at start will be lower.

(c) The starting current is higher.

(d) The losses at starting would be lower.

Ans. a

Exp:



For power factor, $\cos\theta = \frac{R_2}{Z_2} = \frac{R_2}{R_2 + jsX_2}$

At starting $(\cos\theta_2)_{starting} = \frac{R_2}{R_2 + X_2}$

Order of R_2/phase = 0.1 to 0.2 Ω

Order of X_2/phase = 1.5Ω to 2 Ω

$R_2^2 <<< X_2^2$

So, $R_2^2 \rightarrow neglected$

$$\cos\theta_2 = \frac{R_2}{X_2}$$

As R_2 increases, so power factor also increases.

Hence, option (b) is wrong.

$$T_s = \frac{180}{2\pi N_s} \times \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

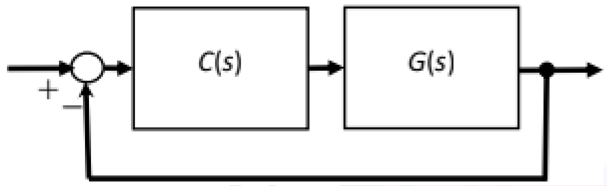
So, R_2^2 term is neglected.

$$T_s = \frac{180}{2\pi N_s} \frac{E_2^2 R_2}{X_2^2}$$

So, starting torque increases as it is directly proportional to rotor resistance.

29. Consider a unity feedback configuration with a plant and a PID controller as shown in the figure.

$G(s) = \dfrac{1}{(s+1)(s+3)}$ and C(s) = K with K being scalar. The closed loop is



(a) only stable for K > 0

(b) only stable for K between -1 and +1

(c) only stable for K < 0

(d) stable for all values of K

Ans. a

Exp:

q(s) = 1 + C(s)G(s) = 0

q(s) = $s^3 + s^2(K + 4) + s(6K + 3) + 10K = 0$

Necessary condition:

$K > -4;\ K > -\frac{1}{2};\ K > 0$

Sufficient condition:

bc > ad (K + 4)(6K + 3) > 10 K

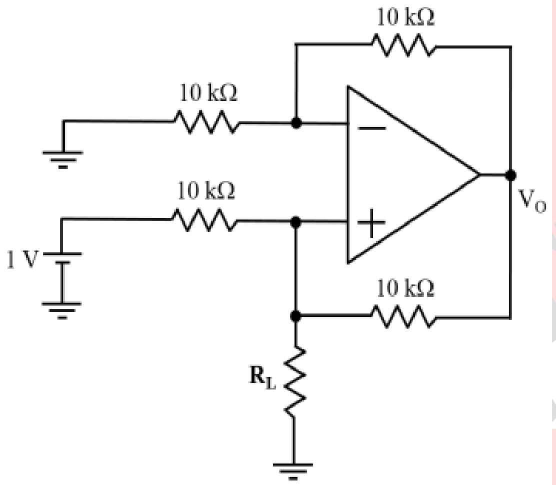
$\Rightarrow \quad 6K^2 + 17K + 12 > 0$

$\Rightarrow \quad$ and $K > \frac{-3}{2}$ or $K < \frac{-4}{3}$ and K < –3

Finally, K > 0 and $K > \frac{-4}{3}$

$\therefore \quad$ The closed loop system is stable only for K > 0.

30. The output V_o of the ideal OpAmp used in the circuit shown below is 5 V. Then the value of resistor RL in kilo ohm (kΩ) is



(a) 2.5

(b) 5

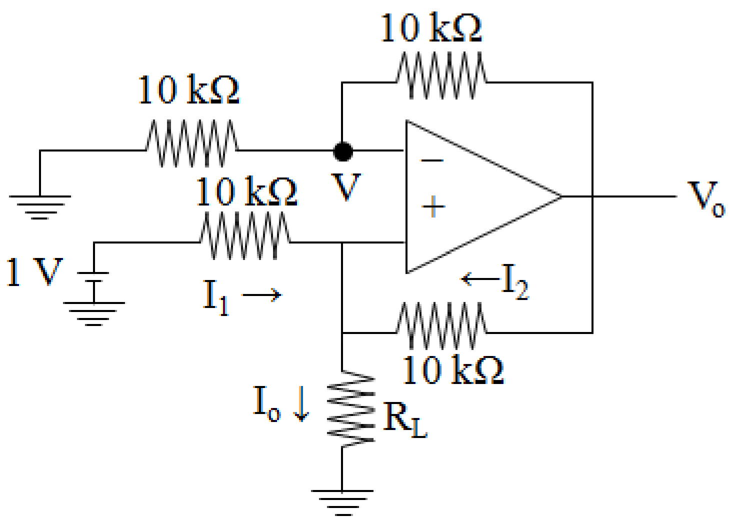
(c) 25

(d) 50

Ans. c

Exp:

Given, V_o = 5 V



$$V = \frac{V_o \times 10}{10+10} = \frac{V_o}{2} = 2.5V$$

Using virtual short property,

$V^+ - V^- = 2.5$ V

By KCL, $I_1 + I_2 = I_o$

$$\frac{1-2.5}{10} + \frac{5-2.5}{10} = \frac{2.5}{R_L}$$

$\Rightarrow$ $R_L = 2.5$ kΩ

31. A Boolean function F of three variables X, Y, and Z is given as

F (X, Y, Z) = (X′ + Y + Z).(X + Y′ + Z′).(X′ + Y + Z′).(X′ Y′ Z′ + X′ Y Z′ + X Y Z′)

Which one of the following is true?

(a) F (X, Y, Z) = (X + Y + Z′).(X′ + Y′ +Z′)

(b) F (X, Y, Z) = (X′ + Y).(X + Y ′+ Z′)

(c) F (X, Y, Z) = X′ Z′ + Y Z′

(d) F (X, Y, Z) = X′ Y′ Z + X Y Z

Ans. c

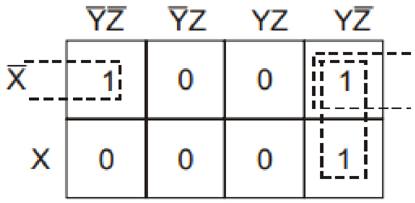
Exp:

(X'Y'Z' + X'Y'Z' + XYZ') = X'Z' (Y' + Y) + XYZ'

= X'Z' + XYZ'

= Z' (X' + XY)

= Z' (X' + Y)



By using k-map, F = X'Z' + Y'

32. A $10\frac{1}{2}$ digit Counter-timer is set in the 'frequency mode' of operation (with T_s = 1 s). For a specific input, the reading obtained is 1000. Without disconnecting this input, the Counter-timer is changed to operate in the 'Period mode' and the range selected is microseconds (μs, with fs = 1 MHz). The counter will then display

(a) 0

(b) 10

(c) 100

(d) 1000

Ans. d

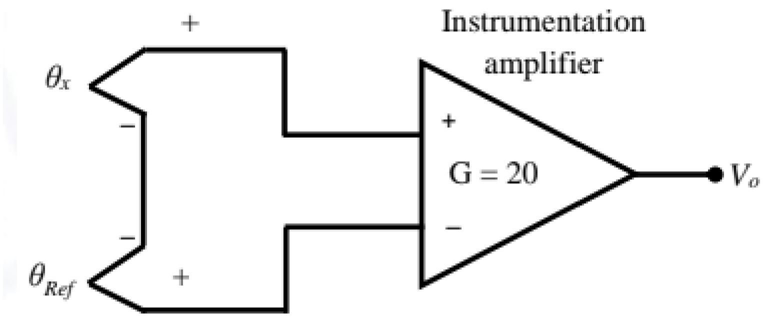
Exp:

When scale set on 1s, reading = 1000

Time period = $\frac{1}{1000} = 10^{-3}$ sec

If new range is 1 MHz, Reading = $\frac{10^{-3}}{10^{-6}}$ 1000

33. A J-type thermocouple has an output voltage $V_\theta = (13650 + 50\ \theta_x)$ μV, where θ_x is the junction temperature in Celsius (°C). The thermocouple is used with reference junction compensation, as shown in the figure. The Instrumentation amplifier used has a gain G = 20. If θ_{Ref} is 1 °C, for an input θ_x of 100 °C, the output V_o of the instrumentation amplifier in millivolt is



(a) 98 mV

(b) 99 mV

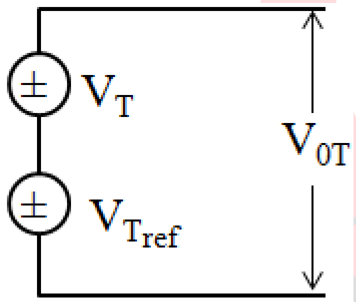
(c) 100 mV

(d) 101 mV

Ans. b

Exp:

$V_{0T} = V_T - V_{Tref} = [13650 + 50\ \theta_x] - [13650 + 50\ \theta_{ref}]$

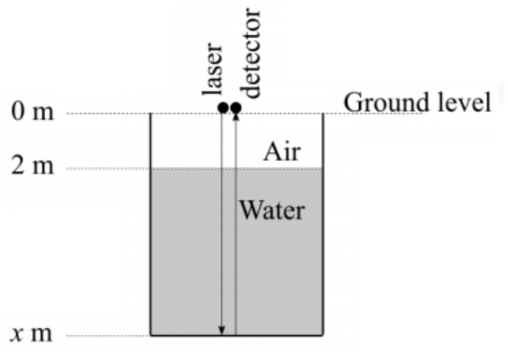


$= 50\ [\theta_x - \theta_{ref}] = 50\ [100 - 1] = 4950$

$V_0 = 20 \times V_{0T} = 4950 \times 20\ \mu V = 99\ mV$

34. A laser pulse is sent from ground level to the bottom of a concrete water tank at normal incidence. The tank is filled with water up to 2 m below the ground level. The reflected pulse from the bottom of the tank travels back and hits the detector. The round-trip time elapsed between sending the laser pulse, the pulse hitting the bottom of the tank, reflecting back and sensed by the detector is 100 ns. The depth of the tank from ground level marked as x in metre is ____.

(Refractive index of water $n_{water} = 1.3$ and velocity of light in air $c_{air} = 3 \times 10^8$ m/s)



(a) 9

(b) 10

(c) 11

(d) 12

Ans. d

Exp:

$t_{journey} = t_1 + t_2 + t_3 + t_4$

$$= \frac{2}{c} + 2t_2 + \frac{2}{c} \qquad \left[t_2 = t_3 = \frac{h}{v} = \frac{1.3Ch}{c}\right]$$

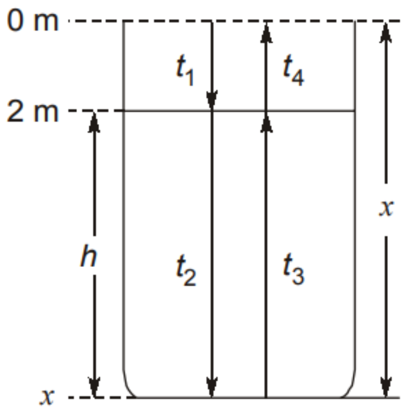
$$= \frac{4}{c} + \frac{2 \times 1.3h}{c}$$

$$100\,\text{ns} = \frac{4 + 2.6h}{c}$$

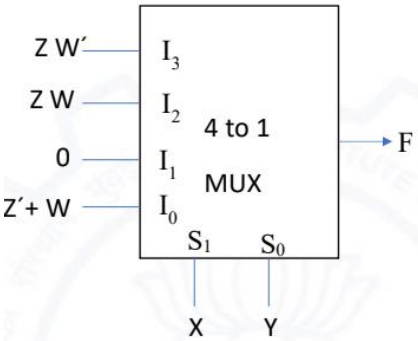
$100 \times 10^{-9} \times 3 \times 10^{8} = 4 + 2.6$ h

$\Rightarrow$ h = 10

$\therefore$ x = (10 + 2)m = 12 m



35. A 4 × 1 multiplexer with two selector lines is used to realize a Boolean function F having four Boolean variables X, Y, Z and W as shown below. S0 and S1 denote the least significant bit (LSB) and most significant bit (MSB) of the selector lines of the multiplexer respectively. I0, I1, I2, I3 are the input lines of the multiplexer.



The canonical sum of product representation of F is

(a) F (X, Y, Z, W) = Σ m(, ,3, 4, 5)

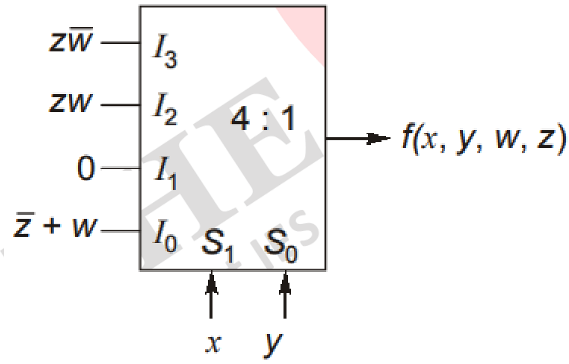
(b) F (X, Y, Z, W) = Σ m(, ,3,11,14)

(c) F (X, Y, Z, W) = Σ m(,5,9, , 4)

(d) F (X, Y, Z, W) = Σ m(1,3,7,9,15)

Ans. b

Exp:



$$f = \bar{x}\,\bar{y}\left(\bar{z}+w\right)+\bar{x}y\cdot 0+x\bar{y}\left(zw\right)+xy\left(z\bar{w}\right)$$

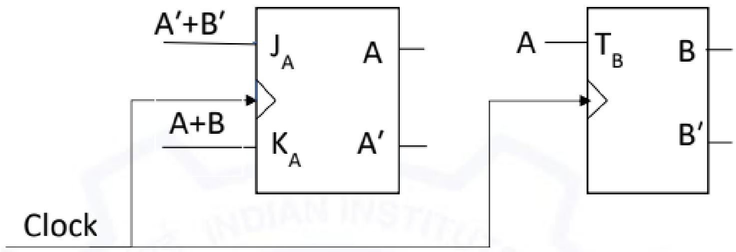
$\bar{x}\ \bar{y}\ \bar{z}\ -\ +\ \bar{x}\ \bar{y}\ -\ w\ +\ x\ \bar{y}\ z\ w\ +\ x\ y\ z\ \bar{w}$

0 0 0 0   0 0 0 1   1 0 1 1   1 1 1 0

0 0 0 1   0 0 1 1

f(x, y, z, w) = Σm(0, 1, 3, 11, 14)

36. Given below is the diagram of a synchronous sequential circuit with one J-K flip-flop and one T flip-flop with their outputs denoted as A and B respectively, with JA = (A′ + B′), KA = (A + B), and T_B = A.



Starting from the initial state (AB = 00), the sequence of states (AB) visited by the circuit is

(a) 00 → 01 → 10 → 11 → 00 ...

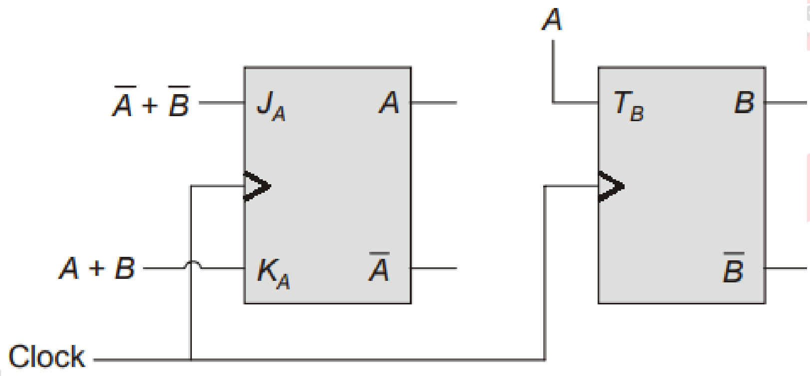
(b) 00→ 10→ 01 → 11 → 00 ...

(c) 00 → 10→ 11 → 01 → 00...

(d) 00 → 01 → 11 → 00 ...

Ans. b

Exp:



$A^+ = J\bar{A} + \bar{K}A$

$B^+ = T_B \oplus B$

$A^+ = (\bar{A}+\bar{B})\cdot\bar{A} + (\overline{A+B})\cdot A$

$B^+ = A \oplus B$

$A^+ = \bar{A}$

	$\bar{A}$	$A \oplus B$
$A\ B$	A^+	B^+
0 0	1	0
1 0	0	1
0 1	1	1
1 1	0	0

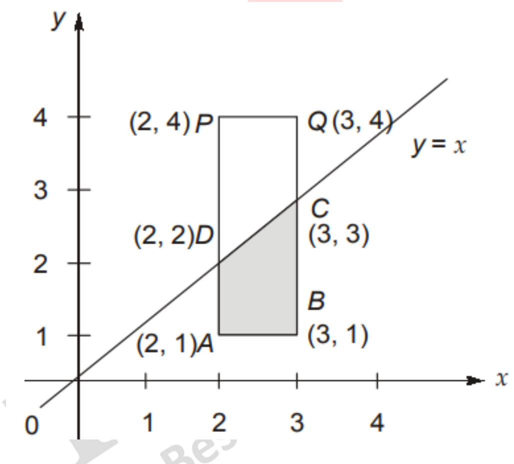
00→ 10→ 01 → 11 → 00 ...

Q.37 – Q.55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).

37. Consider that X and Y are independent continuous valued random variables with uniform PDF given by X ~ U(2, 3) and Y ~ U(1, 4). Then P(Y ≤ X) is equal to __________ (rounded off to two decimal places).

Ans. 0.45 – 0.55

Exp:



Total area = AB × AP = 1 × 3 = 3

Favourable Area (F_{av}) = Area of ABCD (i.e., Trapezium)

$= \frac{1}{2}$(sum of parallel sides) × (Distance between them)

$= \frac{1}{2}(AD + BC) \times (AB) = \frac{1}{2}(1 + 2) \times 1 = \frac{3}{2}$

$$P(y \le n) = \frac{F_{av.}\ area}{Total\ area} = \frac{3/2}{3} = 0.5$$

38. Given A = $\begin{pmatrix} 2 & 5 \\ 0 & 3 \end{pmatrix}$. The value of the determinant $|A^4 - 5A^3 + 6A^2 + 2I|$ = ___.

Ans. 4

Exp:

Characteristic Eq. of A is $|A - \lambda I| = 0$

$\Rightarrow \quad (\lambda - 2)(\lambda - 3) = 0 \Rightarrow \lambda^2 - 5\lambda + 6 = 0$

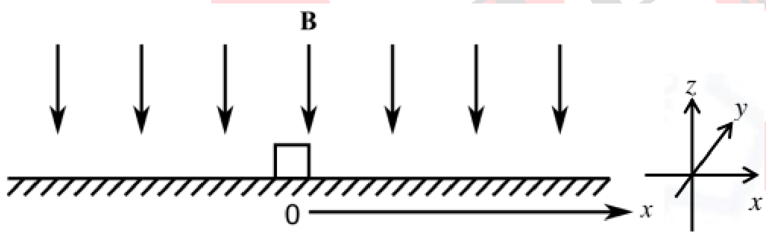
By Cayley Hamilton theorem replace $\lambda \to A$ in Characteristic Eq.

$A^2 - 5A + 6I = 0$

Now $|A^4 - 5A^3 + 6A^2 + 2I| = |A^2(A^2 - 5A + 6I) + 2I|$

$= |0 + 2I| = |2I| = 2^2|I| = 4|I| = 4 \times 1 = 4$

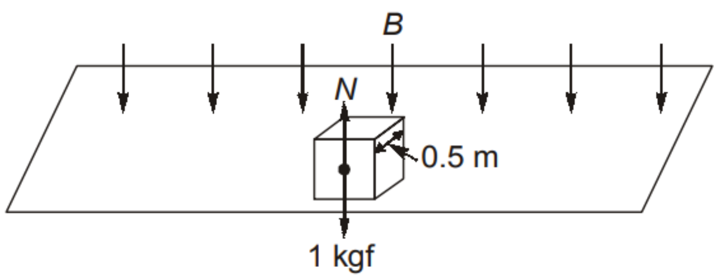
39. The figure below shows an electrically conductive bar of square cross-section resting on a plane surface. The bar of mass of 1 kg has a depth of 0.5 m along the y direction. The coefficient of friction between the bar and the surface is 0.1. Assume the acceleration due to gravity to be 10 m/s^2. The system faces a uniform flux density $B = -1\ \hat{z}$ T. At time t = 0, a current of 10 A is switched onto the bar and is maintained.



When the bar has moved by 1 m, its speed in metre per second is ____(rounded off to one decimal place).

Ans. 2.7 – 2.9

Exp:



Weight of the body = 1 kg

Normal reaction from surface = 1 × 10 = 10 N

Friction force, $F_f = \mu N$

where μ is coefficient of friction of surface = 0.1

∴ $F_f = 10 \times 0.1 = 1$ N (opposing the motion)

Motion of bar caused by force,

F = BIL = (–1) (10)(0.5) = 5 N

Net force on body = 5 – 1 = 4 N (in direction of motion)

Force, F = ma

$4 = 1 \times a \quad \Rightarrow \quad a = 4 \text{ m/s}^2$

Using motion equation: $v^2 = u^2 + 2aS$

u = 0, S = 1 m

$v^2 = 2 \times 4 \times 1 = 8$

$v = \sqrt{8}$ m/s = 2.8 m/s

40. A toroid made of CRGO has an inner diameter of 10 cm and an outer diameter of 14 cm. The thickness of the toroid is 2 cm. 200 turns of copper wire is wound on the core. $\mu_o = 4\pi\times10^{-7}$ H/m and μR of CRGO is 3000. When a current of 5 mA flows through the winding, the flux density in the core in millitesla is ____.

Ans. 10

Exp:

$$\phi = \frac{MMF}{s} = \frac{NI}{\frac{l}{a\mu_o\mu_r}} = \frac{NI}{l}(a\mu_o\mu_r)$$

$$B = \frac{d}{a} = \frac{\frac{NI}{l}(a\mu_o\mu_r)}{a}$$

$$B = \frac{NI\mu_o\mu_r}{l}$$

$$B = \frac{200(5\times10^{-3})(4\pi\times10^{-7})(3000)}{2\pi\left(\frac{12\times10^{-2}}{2}\right)} = 10 \text{ millitesla}$$

41. An air cored coil having a winding resistance of 10 Ω is connected in series with a variable capacitor C_x. The series circuit is excited by a 10 V sinusoidal voltage source of angular frequency 1000 rad/s. As the value of the capacitor is varied, a maximum voltage of 30 V was observed across it. Neglecting skin-effect, the value of the inductance of the coil in millihenry is _____.

Ans. 30

Exp:

As capacitor's varied maximum value of voltage occurs when it is in resonance condition. At resonance condition,

$$I = \frac{V}{R} = \frac{10}{10} = 1\,\text{A}$$

$V_L = V_C = 30$ V

$I\omega L = 30$ V

$\Rightarrow \quad \omega L = 30$

$\therefore \quad L = \frac{30}{1000} = 30$ mH

42. A household fan consumes 60 W and draws a current of 0.3125 A (rms) when connected to a 230 V (rms) ac, 50 Hz single phase mains. The reactive power drawn by the fan in VAr is ____________ (rounded off to the nearest integer).

Ans. 39 – 40

Exp:

As we know that $P_{out} = VI\cos\phi$

$P_{out} = 60$ W

$I = 0.3125$ A

$V = 230$ V

$\cos\phi = 0.8347$

$\phi = 33.406$

$Q = P\tan\phi$

$Q = 39.56$ VAr ≈ 40 VAr

43. Given $y(t) = e^{-3}tu(t) * u(t + 3)$, where * denotes convolution operation.

The value of y(t) as t → ∞ is ___________ (rounded off to two decimal places).

Ans. 0.30 – 0.35

Exp:

$y(t) = e^{-3t} u(t) * u(t + 3)$

$$Y(s) = e^{3s}\left[\frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right)\right]$$

$Y(s) = e^{3s} F(s)$

Where, $F(s) = \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right)$

$$f(t) = \frac{1}{3}\left[1 - e^{-3t}\right] \times u(t)$$

From (i), $Y(s) = e^{3s} F(s)$

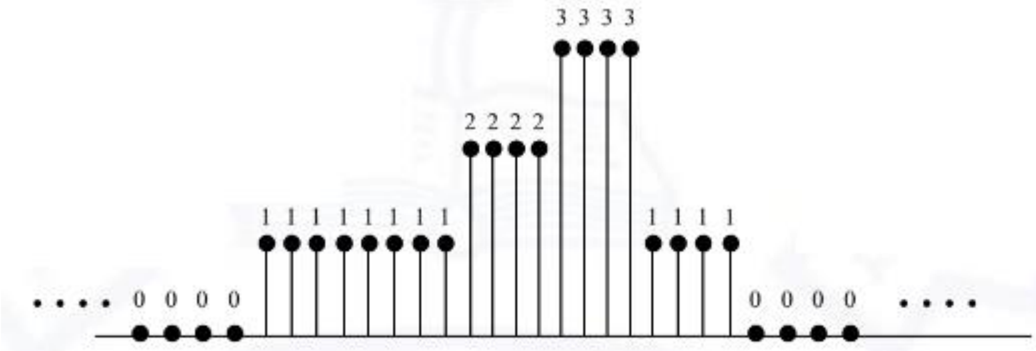
$y(t) = f(t + 3)$

$$y(t) = \frac{1}{3}\left[1 - e^{-3(t+3)}\right]u(t+3)$$

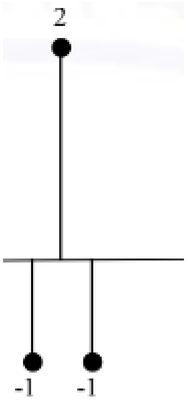
On putting $t = \infty$, we have

$$y(\infty) = \frac{1}{3}[1 - 0] \times 1 = \frac{1}{3}$$

44. The input signal shown below:



is passed through the filter with the following taps :



The number of non-zero output sample is _____.

Ans. 10

Exp:

h(n) \ x(n)	1	1	1	1	1	1	1	1	2	2	2	2	3	3	3	3	1	1	1	1
–1	–1	–1	–1	–1	–1	–1	–1	–1	–2	–2	–2	–2	–3	–3	–3	–3	–1	–1	–1	–1
2	2	2	2	2	2	2	2	2	4	4	4	4	6	6	6	6	2	2	2	2
–1	–1	–1	–1	–1	–1	–1	–1	–1	–2	–2	–2	–2	–3	–3	–3	–3	–1	–1	–1	–1

y[n] = {–1, 1, 0, 0, 0, 0, 0, 0, –1, 1, 0, 0, –1, 1, 0, 0, 2, –2, 0, 0, 1, –1}

Number of non-zero samples in y[n] = 10

45. A sinusoid ($\sqrt{2}$ sin t) μ(t), where μ(t) is the step input, is applied to a system with transfer-function G(s) = $\frac{1}{s+1}$. The amplitude of the steady state output is ____.

Ans. 0.95 – 1.05

Exp:

G(s) = $\frac{1}{s+1}$

r(t) = A sin ωt = $\sqrt{2}$ sint

c(t) = B sin (ωt + ϕ)

B = A|G(jω)|

$$= \sqrt{2}\left|\frac{1}{s+1}\right|_{s=j1} = 1$$

46. Consider a system with transfer-function G(s) = $\frac{2}{s+1}$. A unit step function μ(t) is applied to the system, which results in an output y(t). If e(t) = y(t) − μ(t), then $\lim_{t\to\infty} e(t)$ is _______.

Ans. 1

Exp:

TF = $\frac{2}{s+1}$

OLTF = $\frac{2}{s-1}$

Type = 0

$e_{ss} = \frac{1}{1+k_p}$ for step input,

$$k_p = \lim_{s\to 0}\frac{2}{s-1} = -2$$

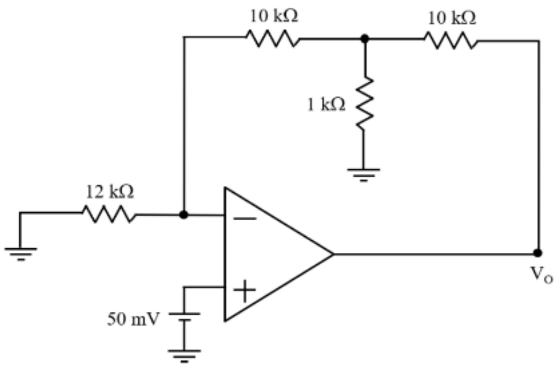
$\therefore \quad e_{ss} = \frac{1}{1-2} = -1$

It is given that e(t) = y(t) − μ(t)

Then,

e_{ss} = 1

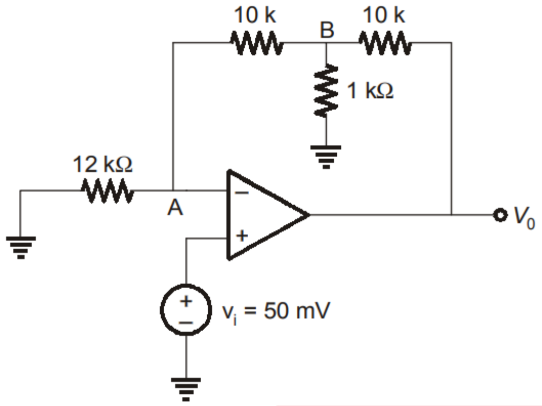
47. The circuit shown below uses an ideal Op-Amp. Output V_O in volt is ________ (rounded off to one decimal place).



Ans. 1.0 – 1.1

Exp:

Using virtual s/c property,



$V^- = V^+$

$\Rightarrow \quad V_A = V_i = 50$ mV

By KCL at A, we get

$$\frac{0-V_A}{12}+\frac{V_B-V_A}{10}=0$$

$$\Rightarrow \quad V_B=\frac{11V_A}{6}=\frac{11V_i}{6}$$

By KCL at A, we get

$$\frac{V_A-V_B}{10}+\frac{0-V_B}{1}+\frac{V_o-V_B}{10}=0$$

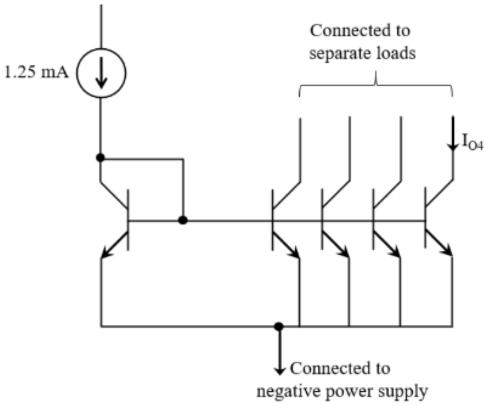
$\Rightarrow \quad V_A - V_B - 10V_B + V_o - V_B = 0$

$V_o = 12\ V_B - V_A$

$$= 12\times\frac{11V_i}{6}-V_i$$

$= 21\ V_i = 21 \times 50$ mV $= 1050$ mV or 1.05 V ≈ 1.1 V

48. All the transistors used in the circuit are matched and have a current gain β of 20. Neglecting the Early effect, the current I_{O4} in milliampere is _____.

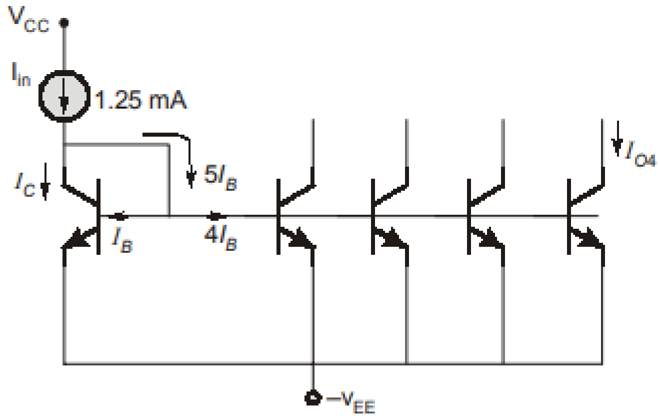


Ans. 1

Exp:

Given that, $\beta = 20$

All transistors are identical and their V_{BE} values are perfectly equal. Hence, their currents will be equal.



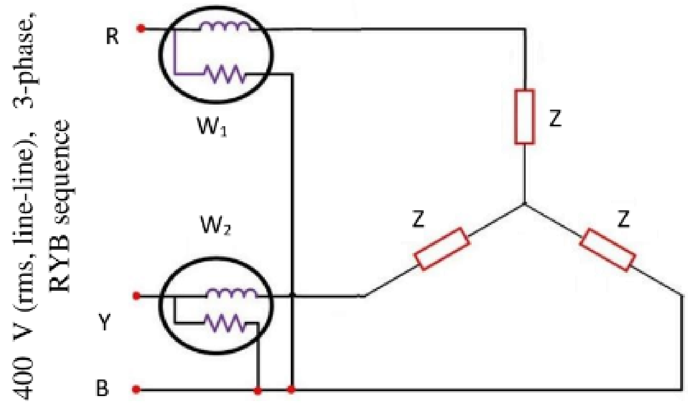
By KCL, $I_{in} = I_C + 5I_B = I_C + \frac{5I_C}{\beta}$

$$I_{in} = I_C + 5I_B = I_C + \frac{5I_C}{\beta}$$

$$I_C = \frac{I_{in}}{1 + \frac{5}{\beta}}$$

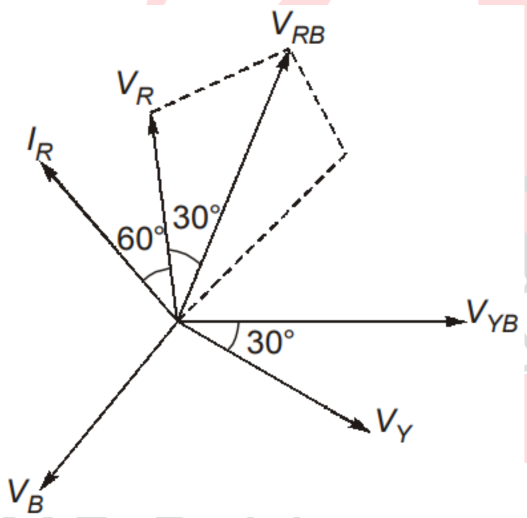
$$I_{o4} = \frac{I_{in}}{1 + \frac{k}{\beta}} = \frac{1.25}{1 + \frac{5}{20}} = 1 \text{ mA}$$

49. The power in a 400 V (rms, line-line) three-phase, three-wire RYB sequence system is measured using the two wattmeters, as shown. The R-line current is 5∠60° A. Wattmeter W1 in the R-line will read (in watt) _________.



Ans. 0

Exp:



As wattmeter W_1 is connected with pressure coil in R-B phase and current coil in R-Phase.

So, Power measured in wattmeter W_1 is = $V_{RB} \times I_R \cos\theta = 400 \times 5 \times \cos 90° = 0$ Watts

50. A $3\frac{1}{2}$ digit, rectifier type digital meter is set to read in its 2000 V range. A symmetrical square wave of frequency 50 Hz and amplitude ±100 V is measured using the meter. The meter will read _____.

Ans. 111

Exp:

As given meter is $3\frac{1}{2}$ digit meter reading will be displayed as 100.0

As input is symmetrical square wave of amplitude 100 V.

51. A bar primary current transformer of rating 1000/1 A, 5VA, UPF has 995 secondary turns. It exhibits zero ratio error and phase error of 30 minutes at 1000 A with rated burden. The watt loss component of the primary excitation current in ampere is __________ (rounded off to one decimal place).

Ans. 4.9 – 5.1

Exp:

n = 995

Ratio error = 0

$k_n - R = 0$

$k_n = R$

$k_n = \frac{1000}{1} = 1000$

So, $R = \frac{1000}{R}$

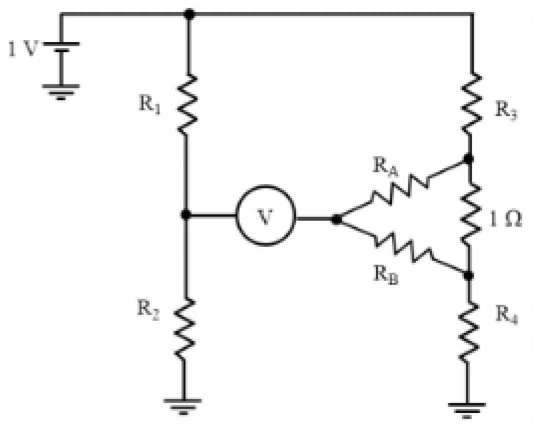
We know that, $R = n + \frac{I_o \sin(\alpha + \delta)}{I_s}$

Here, $\delta = 0$ as UPF

$1000 = 995 + \frac{I_w}{1}$

$I_w = 5$ A

52. In the bridge circuit shown, the voltmeter V showed zero when the value of the resistors are: R_1 = 100 Ω, R_2 = 110Ω, and R_3 = 90Ω. If $(R_1/R_2) = (R_A/R_B)$, the value of R4 in ohm is _____.

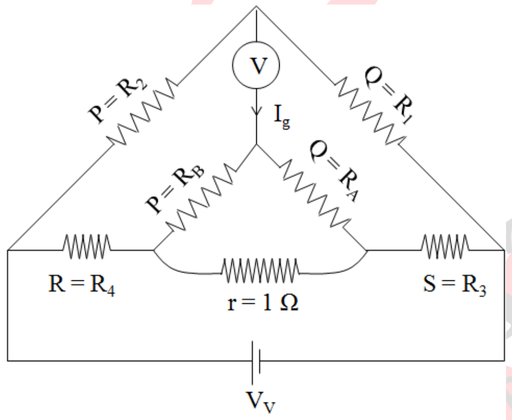


Ans. 99

Exp:

By redrawing Circuit, we can get, "Kelvin Double Bridge"

Given that, $R_1 = 100\ \Omega$, $R_2 = 110\ \Omega$, $R_3 = 90\ \Omega$



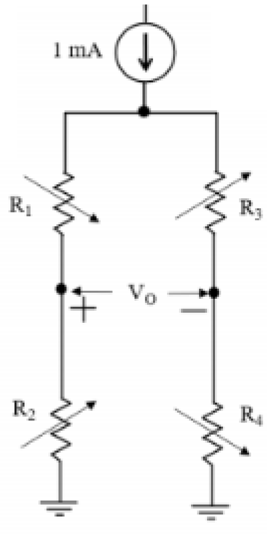
Balance condition, $R = \frac{P}{Q} \times S + \frac{qr}{p+q+r}\left[\frac{P}{Q} - \frac{p}{q}\right]$

By comparing, $\frac{R_1}{R_2} = \frac{R_A}{R_B} \quad \Rightarrow \quad \frac{P}{Q} = \frac{P}{q} \quad \Rightarrow \quad R = \frac{P}{Q} \times S$

$\therefore \quad R_H = \frac{R_2}{R_1} \times R_3 = \frac{110}{100} \times 90$

$R_4 = 99\ \Omega$

53. For the full bridge made of linear strain gages with gauge factor 2 as shown in the diagram, $R_1 = R_2 = R_3 = R_4 = 100\ \Omega$ at 0 °C and strain is 0. The temperature coefficient of resistance of the strain gauges used is 0.005 per °C. All strain gauges are made of same material and exposed to same temperature. While measuring a strain of 0.01 at a temperature of 50 °C, the output V_O in millivolt is _____ (rounded off to two decimal places).



Ans. 2.45 – 2.55

Exp:

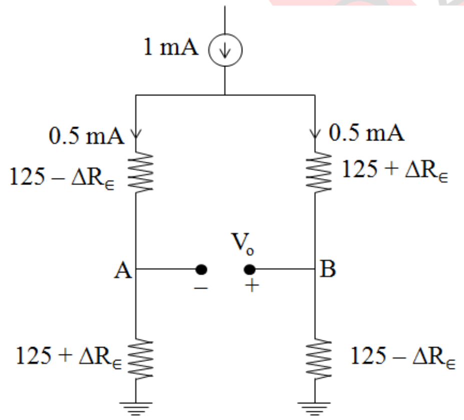
Given data : $G_F = 2$ At T = 0°C, $\in = 0$

$R_1 = R_2 = R_3 = R_4 = 100\ \Omega$, $\alpha = 0.005/°C$

At T = 50°C, $\in = 0.01$ and $V_o = 1$

$R = R_o(1 + \alpha\Delta T) = 100(1 + 0.005 \times 50)$

$R = 125\ \Omega$



Resistance change due to ∈,

$R = 125 \pm \Delta R_\in$

$V_o = 0.5\ mA(125 + \Delta R_\in - 125 + \Delta R_\in)$

$V_o = \Delta R_\in\ mV$

$V_o = R_\in G_F\ mV = 125 \times 0.01 \times 2\ mV = 2.5\ mV$

54. A signal having a bandwidth of 5 MHz is transmitted using the Pulse code modulation (PCM) scheme as follows. The signal is sampled at a rate of 50% above the Nyquist rate and quantized into 256 levels. The binary pulse rate of the PCM signal in Mbits per second is ___.

Ans. 120

Exp:

$f_m = 5$ MHz

N.R = $2f_m = 2 \times$ BW

N.R = 10 MHz

$f_s = 1.5 \times \text{N.R} = 1.5 \times 10 = 15 \text{ MHz} = \frac{1}{T_s}$

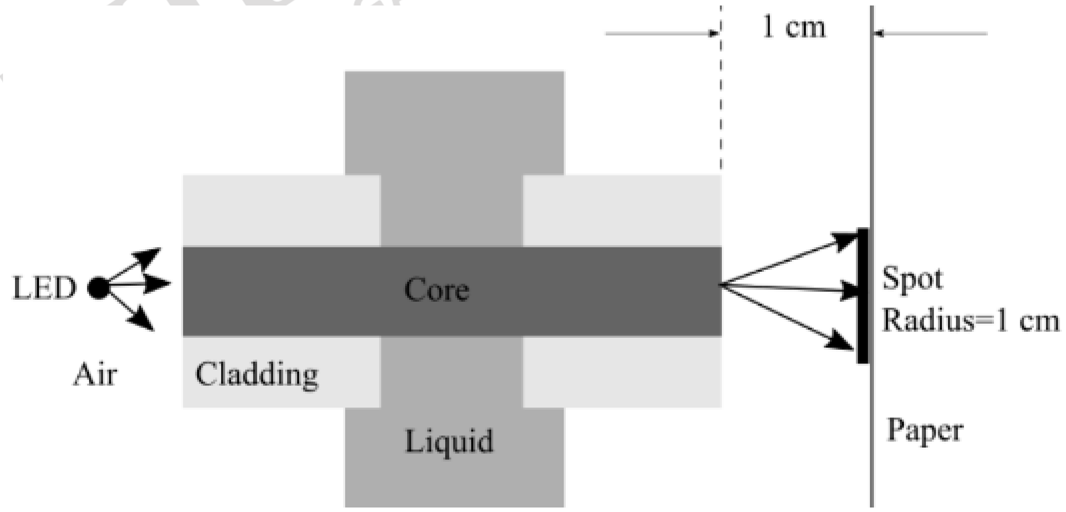
L = 256

$L = 2^n$

So, n = 8 bit/sec

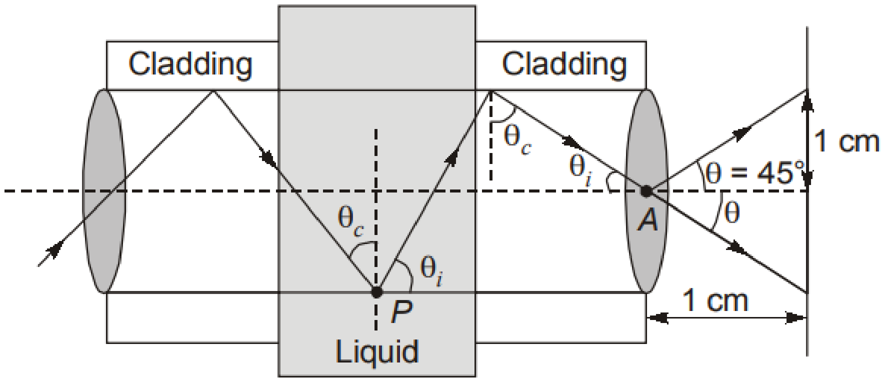
Bit rate, $R_b = \frac{n}{T_s} = 8 \times 15 = 120$ Mbit/sec

55. In the figure shown, a large multimode fiber with $n_{core} = 1.5$ and $n_{clad} = 1.2$ is used for sensing. A portion with the cladding removed passes through a liquid with refractive index n_{liquid}. An LED is used to illuminate the fiber from one end and a paper is placed on the other end, 1 cm from the end of the fiber. The paper shows a spot with radius 1 cm. The refractive index n_{liquid} of the liquid (rounded off to two decimal places) is ___________.



Ans. 1.30 – 1.35

Exp:



Applying Snell's law at point A,

$n_1 \sin\theta_i = n_0 \sin\theta$

$1.5 \sin\theta_i = 1 \times \sin 45°$

$\sin\theta_i = 0.47$

$\theta_i = 28.12°$

$\therefore \quad \theta_c = 90° - \theta_i$

$\theta_c = 61.87°$

Applying Snell's law at point P,

$n_1 \sin\theta_c = n_{liquid}$

$1.5 \sin 61.87 = n_{liquid}$

$\therefore \quad n_{liquid} = 1.32$