GATE 2021

Electrical Engineering (EE)

General Aptitude (GA)

Q.1 Seven cars P, Q, R, S, T, U and V are parked in a row not necessarily in that order. The cars T and U should be parked next to each other. The cars S and V also should be parked next to each other, whereas P and Q cannot be parked next to each other. Q and S must be parked next to each other. R is parked to the immediate right of V. T is parked to the left of U.

Based on the above statements, the only INCORRECT option given below is:

(a) There are two cars parked in between Q and V.

(b) Q and R are not parked together.

(c) V is the only car parked in between S and R

(d) Car P is parked at the extreme end.

Ans. **(a)**

∵ S and V must be parked together and Q and S also must be parked together.

These two condition possiible parkings are:

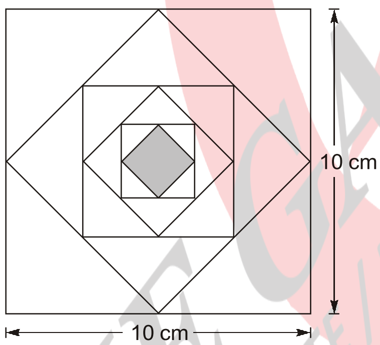
Q S V – – – –

V S Q – – – –

We can observe that in all cases only one car can be parked in between Q and V which is car S.

Hence option (a) is incorrect.

Q.2



In the figure shown above, each inside square is formed by joining the midpoints of the sides of the next larger square. The area of the smallest square (Shaded) as shown, in cm^2 is

(a) 3.125 (b) 12.50

(c) 1.5625 (d) 6.25

Ans. **(a)**

Side of outer first square = 10 cm

Side of outer second square = $\frac{10}{\sqrt{2}}$ cm like wise,

Side of outer third square $= \left(\frac{10}{(\sqrt{2})^2}\right)$

Hence,

side of smallest square $= \frac{10}{(\sqrt{2})^5}$

Area of smallest square $= \left(\frac{10}{(\sqrt{2})^5}\right)^2 = \frac{100}{2^5} = 3.125$

Q.3 Which one of the following numbers is exactly divisible by $(11^{13} + 1)$?

(a) $11^{33} + 1$
(b) $11^{52} - 1$
(c) $11^{26} + 1$
(d) $11^{39} + 1$

Ans. **(b)**

Consider expression $\frac{x^n - a^n}{x + a}$.

Now assuming similar expression $11^{52} - 1 = (11^{13})^4 - (1)^4$

$$\frac{(11^{13})^4 - (1)^4}{11^3 + 1} = \frac{((11)^{13})^2 - (1)^2)((11^{13})^2 + (1)^2)}{11^3 + 1}$$

$$= [(11^{13} - 1)][(11^{13})^2 + (1)^2]$$

Hence, $11^{52} - 1$ is divisible by $11^{13} + 1$.

Q.4 The people ________ were at the demonstration were from all section of society.

(a) whose
(b) who
(c) whom
(d) which

Ans. **(b)**

We cannot use 'which' for people, 'who' is the only word which acts as a subject for the verb.

Q.5 Let X be a continuous random variable denoting the temperature measured. The range of temperature is (0, 100) degree Celsius and let the probability density function of X be $f(x) = 0.01$ for $0 \leq X \leq 100$. The mean of X is _______ .

(a) 5.0
(b) 50.0
(c) 25.0
(d) 2.5

Ans. (b)

n is a continuous random variable for continuous random variable

$$\therefore \qquad f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha \le x \le \beta \\ 0 & \text{otherwise} \end{cases}$$

From here, $\alpha = 0,\ \beta = 100$

For continuous probability function mean

$$= \frac{\alpha + \beta}{2} = \frac{0 + 100}{2} = 50$$

Alternate Solution:

Given: pdf $f(x) = 10^{-2},\ 0 < x < 100$

$$E(x) = \int_0^{100} x f(x) dx = \int_0^{100} x 10^{-2} dx$$

$$= \frac{1}{100} \left. \frac{x^2}{2} \right]_0^{100} = \frac{100}{2} = 50$$

Q.6 For a regular polygon having 10 sides, the interior angle between the sides of the polygon, in degrees, is :

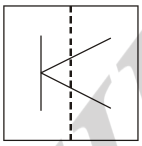
(a) 396
(b) 144
(c) 324
(d) 216

Ans. (b)

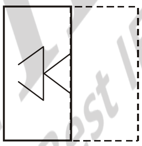
Sum of the interior angle $= (n - 2) \times 180 = 1440$

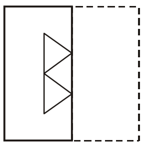
The interior angle between two sides of polygon is $\dfrac{1440}{10} = 144$.

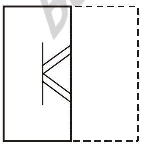
Q.7

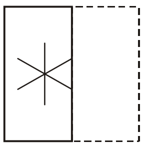


A transparent square sheet above is folded along the dotted line. The folded sheet will look like ______ .

(a)


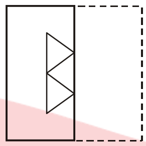
(b)


(c)


(d)


Ans. (b)

Upon rotation, we get the below figure.



Q.8 The importance of sleep is often overlooked by students when they are preparing for exams. Research has consistently shown that sleep deprivation greatly reduces the ability to recall the material learn. Hence, cutting down on sleep to study longer hours can be counterproductive.

Which one of the following statements is the CORRECT inference from the above passage?

(a) Sleeping well alone is enough to prepare for an exam. Studying has lesser benefit.
(b) To do well in an exam, adequate sleep must be part of the preparation.
(c) Students are efficient and are not wrong in thinking that sleep is a waste of time.
(d) If a student is extremely well prepared for an exam, he needs little or no sleep.

Ans. (b)

First statement is ridiculous, third and fourth are contradictory to what is given in the passage.

Q.9 Oasis is to sand as island is to ______ .

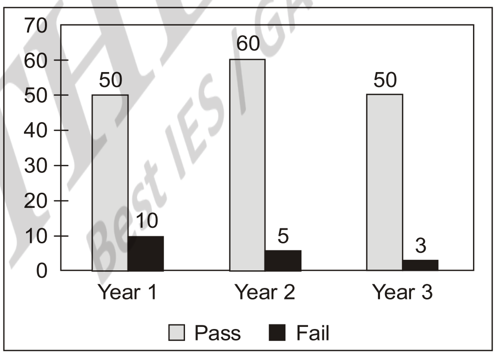
Which one of the following options maintains a similar logical relation in the above sentence?

(a) Mountain
(b) Water
(c) Land
(d) Stone

Ans. (b)

Oasis is surrounded by sand as Island is surrounded by water.

Q.10



The number of students passing or failing in an exam for a particular subject is presented in the bar chart above. Students who pass the exam cannot appear for the exam again. Students who fail the exam in the first attempt must appear for the exam in the following year. Students always pass the exam in their second attempt.

The number of students who took the exam for the first time in the year 2 and the year 3 respectively, are _______ .

(a) 65 and 53
(b) 55 and 53
(c) 60 and 50
(d) 55 and 48

Ans. (d)

Total number of student in the year-2

$= 60 + 5 = 65$

Students who failed in the year and appeared in year-2 = 10

So the students who appeared first time in year-2 = 65 − 10 = 55

Similarly, Total number of students in year-3 = 53

Students who failed in the year-2 and appeared in year-3 = 5

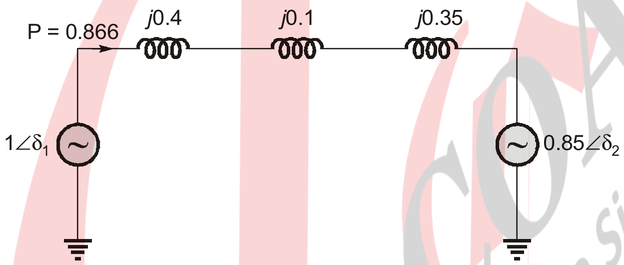
So, the students who appeared first time in year-3

$= 53 - 5 = 48$

Electrical Engineering (EE)

Q.1 An alternator with internal voltage of $1\angle\delta_1$ p.u. and synchronous reactance of 0.4 p.u. is connected by a transmission line of reactance 0.1 p.u. to a synchronous motor having synchronous reactance 0.35 p.u. and internal voltage of $0.85\angle\delta_2$ p.u. If the real power supplied by the alternator is 0.866 p.u, then $(\delta_1 - \delta_2)$ is _____ degrees. (Round off to 2 decimal places)
(Machines are of non-salient type. Neglect resistances)

Ans. **60 (59.50 to 60.50)**



Given real power in (p.u.), $P = 0.866$

$$P = \frac{EV\sin(\delta_1 - \delta_2)}{X_{eq}} = \left|\frac{1\times 0.85}{0.4+0.1+0.35}\right| \sin(\delta_1 - \delta_2)$$

$$(\delta_1 - \delta_2) = 60^\circ$$

Q.2 Lef $f(t)$ be an even function, i.e. $f(-t) = f(t)$ for all t. Let the Fourier transform of $f(t)$ be defined as $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$. Suppose $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω, and $F(0) = 1$. Then

(a) $f(0) = 0$
(b) $f(0) = 1$
(c) $f(0) > 1$
(d) $f(0) < 1$

Ans. **(d)**

$$f(t) \rightleftharpoons F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

The following informations are given about $f(t) \rightleftharpoons F(\omega)$.

(i) $f(t) = f(-t)$

(ii) $F(\omega)\big|_{\omega=0} = 1$

(iii) $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$

From (iii), $\frac{dF(\omega)}{d\omega} + \omega F(\omega) = 0$

By solving the above linear differential equations, (by mathematics)

$$\ln F(\omega) = -\frac{\omega^2}{2}$$

$\Rightarrow$ $$F(\omega) = K \cdot e^{-\omega^2/2} \quad \text{(iv)}$$

Put $\omega = 0$, $F(0) = K$

$\Rightarrow$ $1 = K$ (from info. (ii)

From (iv), $F(\omega) = e^{-\omega^2/2}$

As we know, $e^{-at^2}, a > 0 \rightleftharpoons \sqrt{\frac{\pi}{a}} e^{-\omega^2/4a}$

At $a = \frac{1}{2}$, $e^{-t^2/2} \rightleftharpoons \sqrt{\frac{\pi}{1/2}} \cdot e^{-\omega^2/2}$

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \rightleftharpoons e^{-\omega^2/2} = F(\omega)$$

Thus, $$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

At $t = 0$, $$f(0) = \frac{1}{\sqrt{2\pi}} < 1$$

Q.3 Suppose the circle $x^2 + y^2 = 1$ and $(x-1)^2 + (y-1)^2 = r^2$ intersect each other orthogonally at the point (u, v). Then $u + u$ = _______ .

Ans. (1)

If two curves cut orthogonally then product of slopes = –1

$$x^2 + y^2 = 1$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$(M_1)_{u,v} = \frac{dy}{dx} = \frac{-x}{y} = \frac{-u}{v}$$

$$(x-1)^2 + (y-1)^2 = r^2$$

$$(M_2)_{u,v} = \left(\frac{dy}{dx}\right) = \left(\frac{-2(x-1)}{2(y-1)}\right)_{(u,v)} = \frac{1-u}{v-1}$$

$\because$ $M_1M_2 = -1$

$$\frac{-u}{v} \times \frac{1-u}{v-1} = -1$$

$$-u + u^2 = -v^2 + v$$

$$u^2 + v^2 = v + u$$

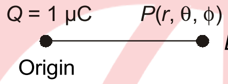
$\because$ $x^2 + y^2 = 1$

$\therefore$ $u + v = 1$

Q.4 A 1 μC point charge is held at the origin of a Cartesian coordinate system. If a second point charge of 10 μC is moved from (0, 10, 0) to (5, 5, 5) and subsequently to (5, 0, 0), then the total work done is _______ mJ. (Round off to 2 decimal places.)

Take $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ in SI units. All coordinates are in meters.

Ans. 9.00 (8.80 to 9.20)



$$\vec{E} = \frac{Q}{4\pi \epsilon_0 r^2}\hat{a}_r = \frac{10^{-6}(9\times 10^9)}{r^2}\hat{a}_r = \frac{9\times 10^3}{r^2}\hat{a}_r$$

In this case work done is independent of type of path but depends an intial and final point.

x	y	z		r	θ	ϕ	
(0,	10,	0)	$\rightarrow$	(10,	90°,	90°)	Initial point
x	y	z		r	θ	ϕ	
(5,	5,	5)	$\rightarrow$	$(5\sqrt{3}$,	54.73°,	45°)	Intermediate point
x	y	z		r	θ	ϕ	
(5,	0,	0)	$\rightarrow$	(5,	90°,	0°)	Final point

Work done (by external source) in moving Q-charge (10 μC) in the presence of electric field $\vec{E}$ is

$$W = -Q\int_{\text{initial point}}^{\text{final point}} \vec{E}\cdot d\vec{l} = -(10\times 10^{-6})\int_{r=10}^{r=5}\frac{9\times 10^3}{r^2}\hat{a}_r \cdot dr\,\hat{a}_r$$

$$W = -10\times 10^{-6}(9\times 10^3)\left(-\frac{1}{r}\right)_{r=10}^{5}$$

$$= 9\times 10^{-2}\left[\frac{1}{5}-\frac{1}{10}\right] = 9\times 10^{-2}\left[\frac{10-5}{50}\right] = 9 \times 10^{-2}\ (10^{-1})$$

$$W = 9 \text{ mJ}$$

Q.5 Consider a power system consisting of N number of buses. Buses in this power system are categorized into slack bus, PV buses and PQ buses for load flow study. The number of PQ buses is N_L. The balanced Newton-Raphson method is used to carry out load flow study in polar form. H, S, M, and R are sub-matrices of the Jacobian matrix J as shown below:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = J\begin{bmatrix}\Delta\delta\\ \Delta V\end{bmatrix}, \text{ where } J = \begin{bmatrix}H & S\\ M & R\end{bmatrix}$$

The dimensions of the sub-matrix is

(a) $N_L \times (N - 1 + N_L)$
(b) $N_L \times (N - 1)$
(c) $(N - 1) \times (N - 1 + N_L)$
(d) $(N - 1) \times (N - 1 - N_L)$

Ans. (b)

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \text{ where } J = \begin{bmatrix} H & S \\ M & R \end{bmatrix}$$

For size of M

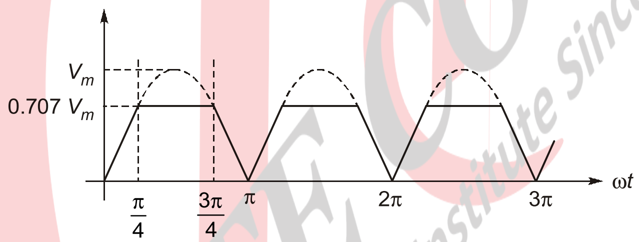
$$\text{Row} = \text{No. of unknown variables of } Q = N_L$$

$$\text{Column} = \text{No. of variable which has } \delta = N_L + (N - 1 - N_L)$$

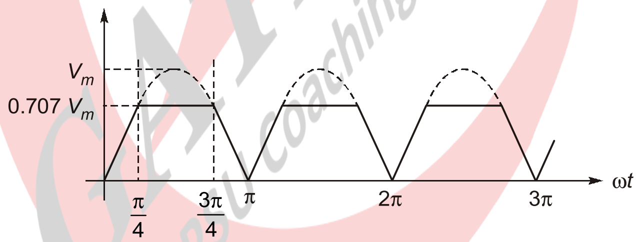
$$= N - 1$$

So, $\text{size of } M = N_L \times (N - 1)$

Q.6 The waveform shown in solid line is obtained by clipping a full-wave rectified sinusoid (shown dashed). The ratio of the RMS value of the full-wave rectified waveform to the RMS value of the clipped waveform is ______ .
(Round off to 2 decimal places.)



Ans. **1.21 (1.19 to 1.24)**



We know,

RMS value of full wave rectified sine wave $= \dfrac{V_m}{\sqrt{2}} = 0.707\ V_m$

For clipped wave form

As $0 \rightarrow \dfrac{\pi}{4}$ and $\dfrac{3\pi}{4} \rightarrow \pi$ wave form are identical

$$\text{rms value of clipped wave} = \sqrt{\frac{1}{\pi}\left[2\int_0^{\pi/4} V_m^2 \sin^2 \omega t + \int_{\pi/4}^{3\pi/4} (0.707V_m)^2 d\omega t\right]}$$

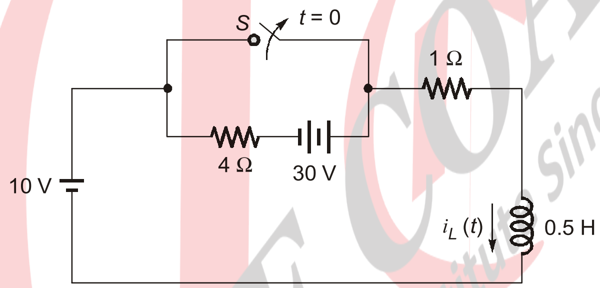
$$= \sqrt{\frac{1}{\pi}\left(\int_0^{\pi/4} \frac{2V_m^2}{2}[1 - \cos 2\omega t]d\omega t + \frac{1}{2}V_m^2 \times \frac{\pi}{2}\right)^{1/2}}$$

$$= \left[\frac{1}{\pi}\left[V_m^2\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) + \frac{V_m^2}{4}\pi\right]\right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi}\left[\frac{\pi}{4} - \frac{1}{2}\right] + \frac{V_m^2}{4}\right]^{1/2} = 0.5838\ V_m$$

$$\text{Ratio} = \frac{0.707}{0.5838} = 1.21$$

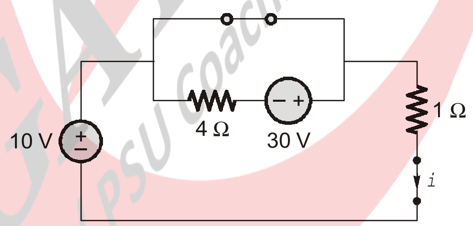
Q.7 In the circuit, switch 'S' is in the closed position for a very long time. If the switch is opened at time t = 0, then $i_L(t)$ in amperes, for $t \geq 0$ is



(a) 10
(b) $8e^{-10t}$
(c) $8 + 2e^{-10t}$
(d) $10(1 - e^{-2t})$

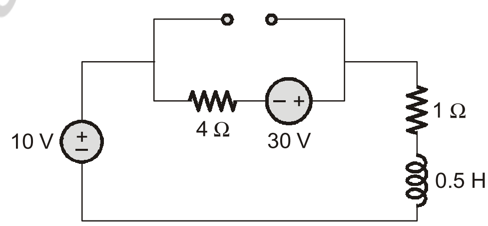
Ans. **(c)**

At $t = 0^-$

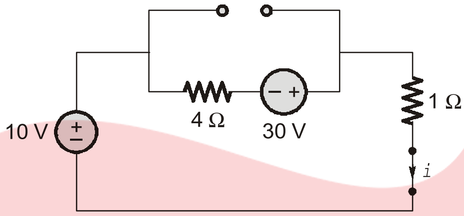


$$i_L(0^-) = \frac{10}{1} = 10\ \text{A}$$

For $t > 0$

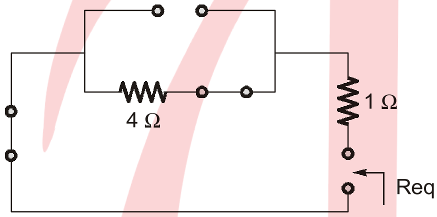


At $t = \infty$



$$i(\infty) = \frac{40}{5} = 8 \text{ A}$$

R_{eq} :



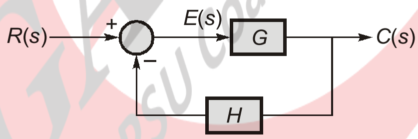
$$R_{eq} = 5\ \Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{0.5}{5} = 0.1 \text{ sec}$$

$$i(t) = 8 + [10 - 8]e^{-t/0.1}$$

$$= 8 + 2e^{-10t} \text{ A}$$

Q.8 For the closed-loop system shown, the transfer function $\frac{E(s)}{R(s)}$ is



(a) $\frac{1}{1+G}$

(b) $\frac{1}{1+GH}$

(c) $\frac{GH}{1+GH}$

(d) $\frac{G}{1+GH}$

Ans. (b)

$$\frac{E(s)}{R(s)} = \frac{R(s) - H \times C(s)}{R(s)} = 1 - H \times \frac{C(s)}{R(s)}$$

$$= 1 - \frac{HG}{1+GH} = \frac{1+GH-GH}{1+GH}$$

$$= \frac{1}{1+GH}$$

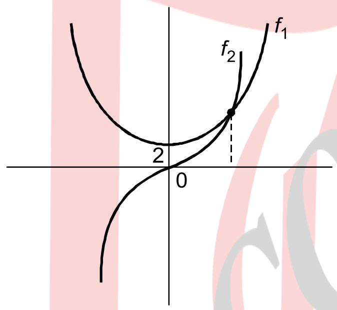
Q.9 In the open interval (0, 1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

(a) three real roots
(b) two real roots
(c) one real roots
(d) no real roots

Ans. (c)

$$x^4 + 2 = 4x^3$$
$$f_1(x) = x^4 + 2$$
$$f_2(x) = 4x^3$$

It is clear that point of intersection of these graphs is solution (or) root of $p(x) = 0$



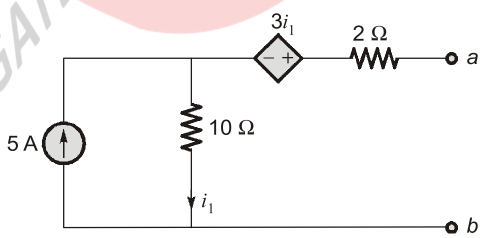
According to intermediate value theorem
$P(0)$ and $P(1)$ are having opposite signs
$\therefore$ a root of $p(x) = 0$ in (0, 1)
and also from graph, there is only one point of intersection
Hence exactly one real root exists in (0, 1)

Q.10 Inductance is measured by

(a) Maxwell bridge
(b) Wien bridge
(c) Schering bridge
(d) Kelvin bridge

Ans. (a)

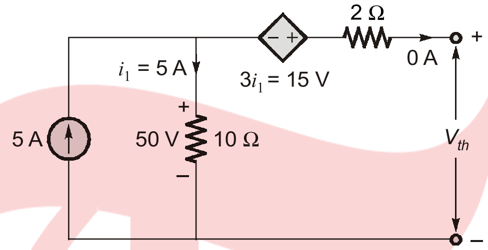
Q.11 For the network shown, the equivalent Thevenin voltage and Thevenin impedance as seen across terminals '*ab*' is



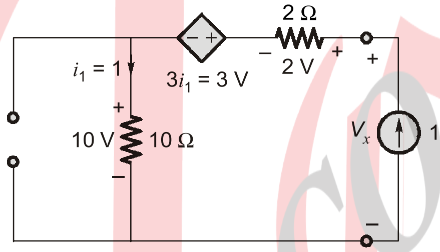
(a) 10 V in series with 12 Ω
(b) 35 V in series with 2 Ω
(c) 50 V in series with 2 Ω
(d) 65 V in series with 15 Ω

Ans. (d)

Given circuit can be resolved as shown below,



$$V_{TH} = 15 + 50 = 65\ V$$

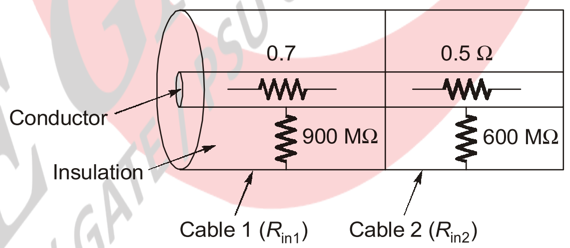


$$V_x = 2 + 3 + 10 = 15\ V$$

$$R_{TH} = \frac{V_x}{1} = 15\ \Omega$$

Q.12 Two single-core power cables have total conductor resistances of 0.7 Ω and 0.5 Ω, respectively, and their insulation resistances (between core and sheath) are 600 MΩ and 900 MΩ. respectively, When the two cables are joined in series, the ratio of insulation resistance to conductor resistance is _______ × 10^6.

Ans. (300)



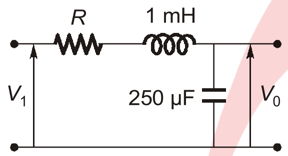
$$(R_{eq})_{conductor} = 0.7 + 0.5 = 1.2\ \Omega$$

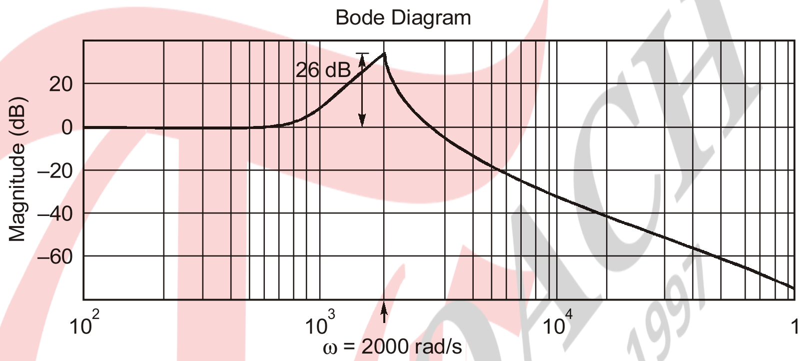
$$(R_{eq})_{in} = \frac{1}{(R_{eq})_{in}} = \frac{1}{(R_{in})_1} + \frac{1}{(R_{in})_2} = \frac{1}{900\times10^6} + \frac{1}{600\times10^6}$$

$$(R_{eq})_{in} = 360 \times 10^6\ \Omega$$

$$\frac{(R_{eq})_{in}}{(R_{eq})_{conductor}} = \frac{360\times10^6}{1.2} = 300 \times 10^6.$$

Q.13 The Bode magnitude plot for the transfer function $\frac{V_0(s)}{V_i(s)}$ of the circuit is as shown. The value of R is _______ Ω. (Round off 2 decimal places)





Ans. **0.10 (0.09 to 0.11)**

From response plot

$$M_r = 26 \text{ dB} = 20$$

$$\therefore \quad \frac{1}{2\xi\sqrt{1-\xi^2}} = 20$$

$$\therefore \quad \xi = 0.025$$

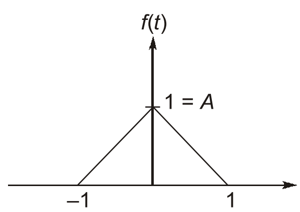
From electrical network

$$\chi = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.025$$

$$\therefore \quad R = 0.10 \ \Omega$$

Q.14 Consider a continuous-time signal $x(t)$ defined by $x(t) = 0$ for $|t| > 1$, and $x(t) = 1-|t|$ for $|t| \le 1$. Let the Fourier transform of $x(t)$ be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$. The maximum magnitude of $X(\omega)$ is ______ .

Ans. **(1)**



Fourier transform, $F(\omega) = A\tau Sa^2\left(\frac{\omega\tau}{2}\right)$

As $A = 1$, $\tau = 1$,

$$F(\omega) = Sa^2\left(\frac{\omega}{2}\right)$$

$$F(\omega)|_{peak} = F(0) = Sa^2(0) = 1$$

$\because$ Peak value of sampling function occurs at $\omega = 0$,

Peak value = 1

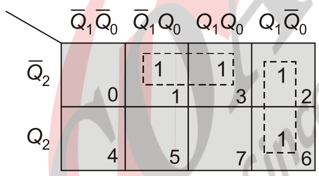
Q.15 A counter is constructed with three D flip-flops. The input-output pairs are named (D_0, Q_0), (D_1, Q_1) and (D_2, Q_2), where the subscript 0 denotes the least significant bit. The output sequence is desired to be the Gray-code sequence 000, 001, 011, 010, 110, 111, 101 and 100, repeating periodically. Note that the bits are listed in the $Q_2Q_1Q_0$ format. The combinational logic expression for D_1 is

(a) $Q_2Q_1 + \bar{Q}_2\bar{Q}_1$ (b) $Q_2Q_1Q_0$

(c) $\bar{Q}_2Q_0 + Q_1\bar{Q}_0$ (d) $Q_2Q_0 + Q_1\bar{Q}_0$

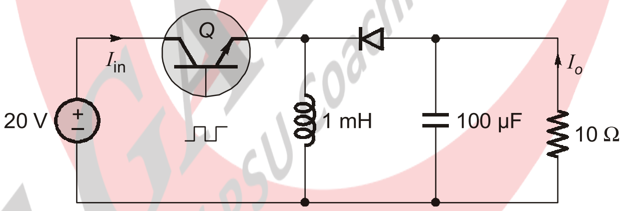
Ans. **(c)**

Present state			Next state					
Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	D_2	D_1	D_0
0	0	0	0	0	1		0	
0	0	1	0	1	1		1	
0	1	1	0	1	0		1	
0	1	0	1	1	0		1	
1	1	0	1	1	1		1	
1	1	1	1	0	1		0	
1	0	1	1	0	0		0	
1	0	0	0	0	0		0	

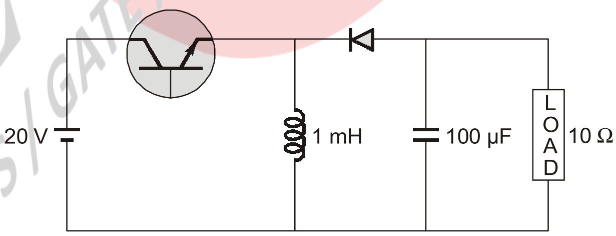


$$D_1 = \bar{Q}_2Q_0 + Q_1\bar{Q}_0$$

Q.16 Consider the buck-boost converter shown. Switch Q is operating at 25 kHz and 0.75 duty-cycle. Assume diode and switch to be ideal. Under steady-state condition, the average current flowing through the inductor is ______ A.



Ans. **(24)**



$$\alpha = 0.75, \quad f = 25 \text{ kHz}$$

Assume continous conduction:

$$V_0 = \frac{\alpha V_s}{1-\alpha} = \frac{0.75 \times 20}{1 - 0.75}$$

$$V_0 = 60 \text{ V}$$

$$I_0 = \frac{V_0}{R} = \frac{60}{10} = 6\text{ A}$$

$$I_L = \frac{I_0}{1-\alpha} = \frac{6}{1-0.75} = 24\text{ A}$$

$$\Delta I_L = \frac{\alpha V_s}{f_L} = \frac{0.75\times 60}{25\times 10^3 \times (1\times 10^{-3})} = 1.8\text{ A}$$

$$I_{L\,min} = I_L - \frac{\Delta I_L}{2} = 24 - \frac{1.8}{2} = 24 - 0.9$$

$(I_{L\,min} = 23.1\text{ A}) > 0$

∴ Continous conduction assumption is correct.

$I_L = 24$ A

Q.17 A belt-driven DC shunt generator running at 300 RPM delivers 100 kW to a 200 V DC grid. It continues to run as a motor when the belt breaks, taking 10 kW from the DC grid. The armature resistance is 0.025 Ω, field resistance is 50 Ω, and brush drop is 2 V. Ignoring armature reaction, the speed of the motor is _______ RPM. (Round off 2 decimal places)

Ans. 275.18 (274.00 to 276.00)

$$I_L = \frac{100\times 10^3}{200} = 500\text{ A}$$

$$I_{sh} = \frac{200}{50} = 4\text{ A}$$

$I_a = 504$ A

$E_g = V + I_a R_a$ + Brush drop

$= 200 + 504\,(0.025) + 2$V

$= 214.6$ V

In motoring case: $VI = 10$ kW, $V = 200$ V

$$\therefore \quad I = \frac{10000}{200} = 50A$$

$I_f = 4$A, $I_a = I_L - I_f$

$= 46$ A

$E_b = V - I_a R_a$ − Brush Drop

$= 200 - 46\,(0.025) - 2 = 196.85$ V

$$\frac{N_m}{N_g} = \frac{E_b}{E_g}$$

$$N_m = \frac{E_b}{E_g}\times N_g$$

$$\therefore \quad N_m = \frac{196.85}{214.6}\times 300 = 275.18\text{ rpm}$$

Q.18 Let $p(z) = z^3 + (1 + j)z^2 + (2 + j)z + 3$, where z is a complex number. Which one of the following is true?

(a) The complex roots of the equation $p(z) = 0$ come in conjugate pairs.
(b) All the roots cannot be real.
(c) The sum of the roots of $p(z) = 0$ is real number
(d) Conjugate $\{p(z)\} = p\{\text{conjugate}(z)\}$ for all z

Ans. (b)

Since sum of the roots is a complex number
$\Rightarrow$ absent one root is complex
So all the roots cannot be real.

Q.19 A signal generator having a source resistance of 50 Ω is set to generate a 1 kHz sinewave. Open circuit terminal voltage is 10 V peak-to-peak. Connecting a capacitor across the terminals reduces the voltage to 8 V peak-to-peak. The value of this capacitor is ______ μF. (Round off to 2 decimal places).

Ans. 2.38 (2.30 to 2.45)

$$\frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\omega RC}$$

$$\frac{8}{10}\angle\theta = \frac{1}{1 + j\omega RC}$$

$$1 + j\omega RC = 1.25\angle\theta$$

$$\omega RC = \sqrt{1.25^2 - 1^2}$$

$$\omega RC = 0.75$$

$$C = \frac{0.75}{2000\pi \times 50} = 2.38\ \mu\text{F}$$

Q.20 The power input to a 500 V, 50 Hz. 6-pole. 3-phase induction motor running at 975 RPM is 40 kW. The total stator losses are 1 kW. If the total friction and windage losses are 2.025 kW. then the efficiency is ________ %.

Ans. (90)

$$P_{i/p} = 40 \text{ kW}, \quad \text{stator loss} = 1 \text{ kW}, \quad F \text{ and } W = 2.025 \text{ kW}$$

$$\text{Stator o/p} = 40 - 1 = 39 \text{ kW}$$

$$\text{Slip} = \frac{1000 - 975}{1000} = 0.025$$

$$\text{Rotor o/p} = \text{Rotor i/p} \times (1 - s)$$

$$= 39\ (1 - 0.025) = 38.025 \text{ kW}$$

$$\text{Motor o/p} = 38.025 - 2.025 = 36 \text{ kW}$$

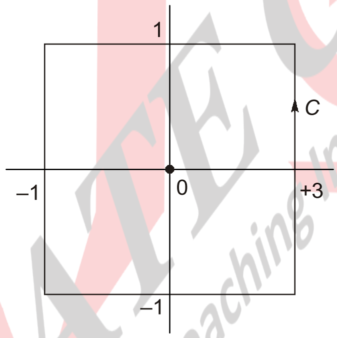
$$\eta = \frac{\text{Motor output}}{\text{Motor input}} = \frac{36}{40} \times 100 = 90\%$$

Q.21 Let $(-1-j)$, $(3-j)$, $(3+j)$ and $(-1+j)$ be the vertices of rectangle C in the complex plane. Assuming that C is traversed in counter-clockwise direction, the value of the contour integral $\oint_C \frac{dz}{z^2(z-4)}$ is

(a) 0
(b) $\frac{-j\pi}{8}$
(c) $\frac{j\pi}{2}$
(d) $\frac{j\pi}{16}$

Ans. **(b)**

$$\oint_C \frac{dz}{z^2(z-4)}$$



Singularities are given by $z^2(z-4) = 0$

$\Rightarrow$ $z = 0, 4$

$z = 0$ is pole of order m = 2 lies inside contour 'c'

$z = 4$ is pole of order m = 1 lies outside 'c'

$$\text{Res}_0 = \frac{1}{(2-1)!} \lim_{z\to 0} \frac{d^{2-1}}{dz^{2-1}}\left[(z-0)^2 \frac{1}{z^2(z-4)}\right]$$

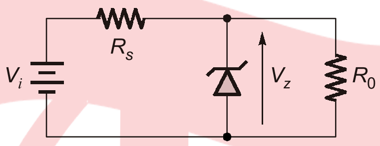
$$= \frac{-1}{(0.4)^2} = \frac{-1}{16}$$

By CRT

$$\oint_C f(z)dz = 2\pi i \text{Res}_0 = 2\pi i\left[\frac{-1}{16}\right]$$

$$= \frac{-\pi i}{8}$$

Q.22 In the circuit shown, a 5 V Zener diode is used to regulate the voltage across load R_0. The input is an unregulated DC voltage with a minimum value of 6 V and a maximum value of 8 V. The value of R_s is 6 Ω. The Zener diode has a maximum rated power dissipation of 2.5 W, Assuming the Zener diode to be ideal, the minimum value of R_0 is ________ Ω.



Ans. (30)

To calculate $R_{0\,min}$, we must find $I_{L\,max}$

$$I_{s\,min} = I_z \text{ min} + I_{L\,max}$$

$$\frac{V_{i\,min} - V_z}{R_s} = I_z \text{ min} + I_{L\,max}$$

For ideal zener diode, $I_{z\,min} = 0$

$$\frac{V_{i\,min} - V_z}{R_s} = I_{L\,max}$$

$$\frac{6-5}{6} = I_{L\,max}$$

$$I_{L\,max} = \frac{1}{6}\text{A}$$

$$R_{0\,min} = \frac{V_z}{I_{L\,max}} = \frac{5}{1/6} = 30\ \Omega$$

Q.23 One coulomb of point charge moving with a uniform velocity $10\hat{x}$ m/s enters the region $x \geq 0$ having a magnetic flux density.

$\vec{B} = (10y\hat{x} + 10x\hat{y} + 10\hat{z})$ T .

The magnitude of force on the charge at $x = 0^+$ is ________ N.

($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis and z-axis, respectively)

Ans. (100)

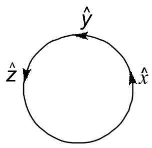
Force on a charge moving with $\vec{v}$ velocity due to magnetic field is

$$\vec{F} = Q(\vec{v} \times \vec{B})$$

$$= 1[10\hat{x} \times (10y\,\hat{x} + 10x\,\hat{y} + 10\,\hat{z}]$$

$$= 10(10x)\hat{z} + 10(10)(-\hat{y})$$

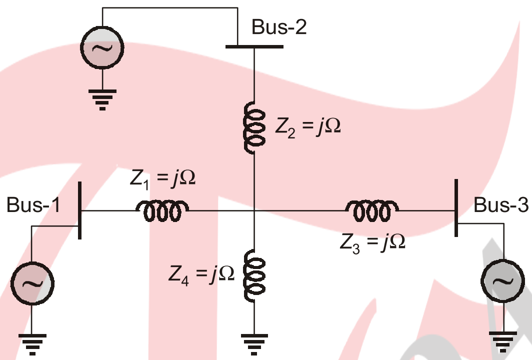
$$= 100x\,\hat{z} - 100\,\hat{y}$$



$$\vec{F}\Big|_{x=0^+} = -100\hat{y}$$

$$|\vec{F}| = \sqrt{(-100)^2} = 100 \text{ Newton}$$

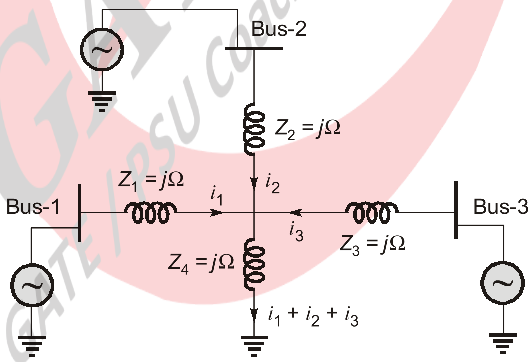
Q.24 A 3 Bus network is shown. Consider generators as ideal voltage sources. If rows 1, 2 and 3 of the Y_{Bus} matrix correspond to Bus 1, 2 and 3 respectively, then Y_{Bus} of the network is



(a) $\begin{bmatrix} -\frac{1}{2}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{1}{2}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{1}{2}j \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{3}{4}j \end{bmatrix}$

(c) $\begin{bmatrix} -4j & j & j \\ j & -4j & j \\ j & j & -4j \end{bmatrix}$ (d) $\begin{bmatrix} -4j & 2j & 2j \\ 2j & -4j & 2j \\ 2j & 2j & -4j \end{bmatrix}$

Ans. **(b)**



$$V_1 = j1i_1 + j1(i_1 + i_2 + i_3)$$

$$V_1 = 2ji_1 + ji_2 + ji_3$$

and

$$V_2 = ji_1 + 2ji_2 + ji_3$$

$$V_3 = ji_1 + ji_2 + 2ji_3$$

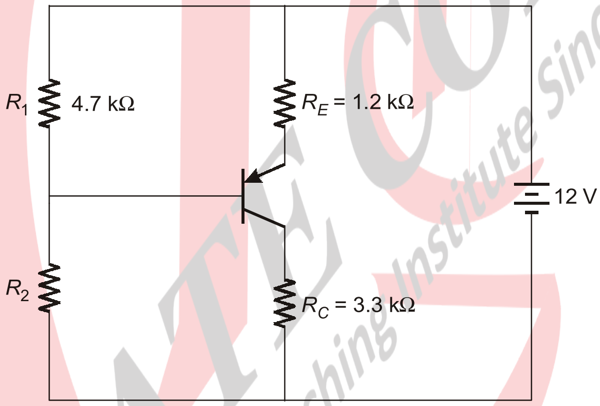
$\therefore$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2j & j & j \\ j & 2j & j \\ j & j & 2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

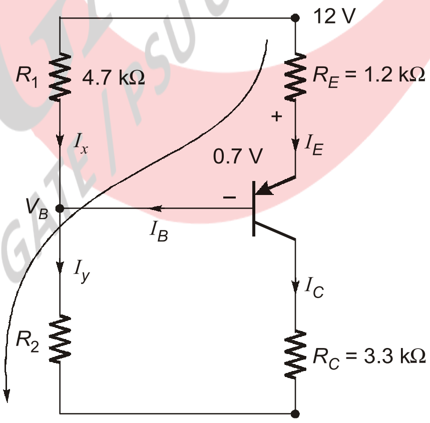
$$Z_{bus} = \begin{bmatrix} 2j & j & j \\ j & 2j & j \\ j & j & 2j \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} \frac{-3j}{4} & \frac{j}{4} & \frac{j}{4} \\ \frac{j}{4} & \frac{-3j}{4} & \frac{j}{4} \\ \frac{j}{4} & \frac{j}{4} & \frac{-3j}{4} \end{bmatrix}$$

Q.25 In the BJT circuit shown, beta of the PNP transistor is 100. Assume V_{BE} = –0.7 V. The voltage across R_C will be 5 V when R_2 is _______ kΩ. (Round off 2 decimal places).



Ans. 17.06 (16.50 to 17.50)



$$I_C = \frac{5V}{3.3k} = 1.515 \text{ mA}$$

$$I_E = 1.53 \text{ mA}$$

$$I_B = 0.0151 \text{ mA}$$

$-12 + 1.2k \times 1.53\ m + 0.7 + V_B = 0$

$$V_B = 9.464\ V$$

$$I_x = \frac{12 - V_B}{4.7k} = \frac{12 - 9.464}{4.7k} = 0.539\ mA$$

$$I_x + I_B = I_y$$

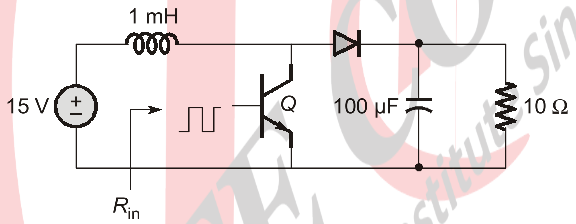
$\Rightarrow$ $$I_y = 0.5396 + 0.0151$$

$$I_y = 0.5546\ mA$$

$$V_B = 0.5546\ m \times R_2 = 9.464$$

$$R_2 = 17.06\ k\Omega$$

Q.26 Consider the boost converter shown. Switch Q is operating at 25 kHz with a duty cycle of 0.6. Assume the diode and switch to be ideal. Under steady-state condition, the average resistance R_{in} as seen by the source is______ Ω. (Round off to 2 decimal places.)



Ans. **1.60 (1.50 to 1.70)**

Checking for continuous conduction mode

$$\Delta I_L = \frac{\alpha V_S}{fL} = \frac{0.6 \times 15}{25 \times 10^3 \times 1 \times 10^{-3}} = 0.36A$$

$$\frac{\Delta I_L}{2} = 0.18A$$

$$I_{L,min} = I_L - \frac{\Delta I_L}{2} = I_S - \frac{\Delta I_L}{2}$$

$$= (9.375 - 0.18) = 9.195 > 0$$

As it is continuous conduction

$$V_0 = \frac{V_S}{1-\alpha} = \frac{15}{1-0.6} = 37.5V$$

$$I_0 = \frac{V_0}{R} = \frac{37.5}{10} = 3.75V$$

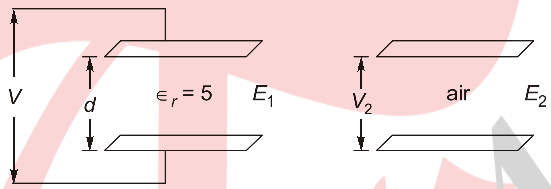
$$\frac{V_0}{V_S} = \frac{I_S}{I_o} = \frac{1}{1-\alpha}$$

$$I_S = \frac{I_o}{1-\alpha} = \frac{3.75}{1-0.6} = 9.375A$$

$$R_{in} = \frac{V_S}{I_S} = \frac{15}{9.375} = 1.6\Omega$$

Q.27 Consider a large parallel plate capacitor. The gap 'd' between the two plates is filled entirely with a dielectric slab of relative permittivity 5. The plates are initially charged to a potential difference of V volts and then disconnected from the source, If the dielectric slab is pulled out completely, then the ratio of the new electric field E_2 in the gap to the original electric field E_1 is ________.

Ans. (5)



If voltage source is removed then in both cases charge Q is constant.

Case-1: $(Q_1, = Q;\ V_1 = V)$

$$Q_1 = C_1 V_1$$

$$Q = \frac{\epsilon_0 (5) A}{d} V_1$$

$$Q = 5\left(\frac{\epsilon_0 A}{d}\right) V_1 \quad \text{...(i)}$$

Case-2: $(Q_2 = Q;\ V_2)$

$$Q_2 = C_2 V_2$$

$$Q = \frac{\epsilon_0 (1) A}{d} V_2$$

$$Q = \left(\frac{\epsilon_0 A}{d}\right) V_2 \quad \text{...(ii)}$$

Equation (i) is equal to equation (ii)

$$\Rightarrow \quad 5\left(\frac{\epsilon_0 A}{d}\right) V_1 = \left(\frac{\epsilon_0 A}{d}\right) V_2$$

$$\Rightarrow \quad 5V_1 = V_2$$

$$\frac{V_2}{V_1} = 5 \quad \text{...(iii)}$$

$$E_1 = \frac{V_1}{d};\ E_2 = \frac{V_2}{d}$$

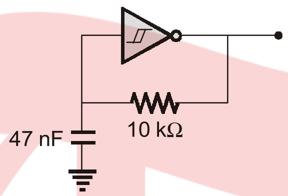
$$\frac{E_2}{E_1} = \frac{V_2 / d}{V_1 / d}$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \frac{V_2}{V_1} \quad \text{...(iv)}$$

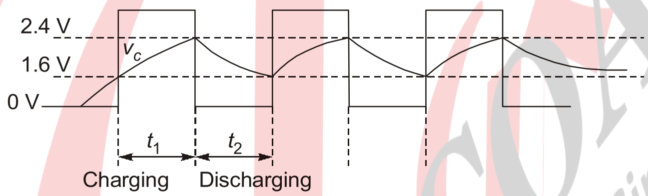
Put equation (iii) in equation (iv),

$$\Rightarrow \quad \frac{E_2}{E_1} = 5$$

Q.28 A CMOS Schmitt-trigger inverter has a low output level of 0 V and a high output level of 5 V. It has input thresholds of 1.6 V and 2.4 V. The input capacitance and output resistance of the Schmitt-trigger are negligible. The frequency of the oscillator shown is _______ Hz. (Round off to 2 decimal places.)



Ans. **3157.56 (3100.00 to 3200.00)**



$$v_c(t) = v_{c\ \text{final}} + [v_{\text{initial}} - v_{c,\ \text{final}}]\ e^{-t/RC}$$

Charging,

$$v_c(t) = 5 + (1.6 - 5)e^{-t/RC}$$
$$= 5 - 3.4\ e^{-t/RC}$$

$t = t_1$,

$$v_c(t) = 2.4\text{ V}$$
$$2.4 = 5 - 3.4\ e^{-t/RC}$$
$$3.4e^{-t1/RC} = 2.6$$
$$t_1 = \ln\left(\frac{3.4}{2.6}\right)RC = 0.268 \times RC$$

Discharging,

$$v_c(t) = 0 + (2.4 - 0)\ e^{-t/RC}$$
$$= 2.4e^{-t/RC}$$

$t = t_2$,

$$v_c(t_a) = 1.6\text{ V}$$
$$1.6 = 2.4e^{-t2/RC}$$
$$t_2 = \ln\left(\frac{2.4}{1.6}\right)RC = 0.405 \times RC$$
$$T = t_1 + t_2 = (0.268 + 0.405)\text{RC}$$
$$T = 0.673\text{ RC}$$
$$f = \frac{1}{0.673RC} = \frac{1}{0.673 \times 10^4 \times 47 \times 10^{-9}}$$
$$f = 3157.46\text{ Hz}$$

Q.29 The state space representation of a first-order system is given as

$$\dot{x} = -x + u$$
$$y = x$$

where, x is the state variable, u is the control input and y is the controlled output. Let $u = -Kx$ be the control law, where K is the controller gain. To place a closed-loop pole at –2, the value of K is _______ .

Ans. (1)

$$\dot{x} = -x - Kx = x(-kI - I)$$

Characteristic equation,

$$|sI + KI + I| = 0$$
$$|(s + 1 + K)I| = 0$$
$$\therefore \quad s + 1 + K = 0$$
$$s = -1 - K$$
$$-2 = -1 - K$$
$$K = 1$$

Q.30 An 8-pole, 50 Hz. three-phase, slip-ring induction motor has an effective rotor resistance of 0.08 Ω per phase. Its speed at maximum torque is 650 RPM. The additional resistance per phase that must be inserted in the rotor to achieve maximum torque at start is _______Ω. (Round off to 2 decimal places.)

Neglect magnetizing current and stator leakage impedance. Consider equivalent circuit parameters referred to stator.

Ans. 0.52 (0.50 to 0.54)

$$N_{max} = 650 \text{ rpm}, \ P = 8, \ 50 \text{ Hz}$$
$$s_m = \frac{750 - 650}{750} = 0.1333$$
$$s_m = \frac{R_2}{X_2}$$
$$\therefore \quad X_2 = \frac{R_2}{s_m} = \frac{0.08}{0.133} = 0.601\,\Omega$$
$$R_2 = 0.08\ \Omega, \ X_2 = 0.601\ \Omega$$

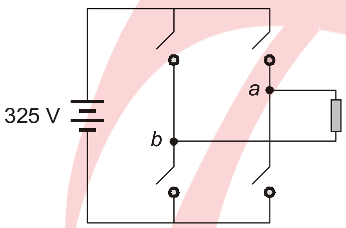
Condition for maximum T_{st}

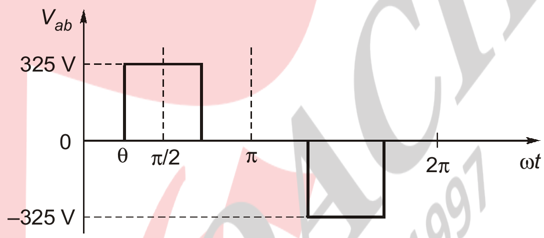
$$\Rightarrow \quad R_2 = X_2$$
$$\therefore \quad R_2 + R_{ext} = X_2$$
$$\therefore \quad R_{ext} = 0.601 - 0.08 = 0.521\ \Omega$$

Q.31 A single-phase full-bridge inverter fed by a 325 V DC produces a symmetric quasi-square waveform across '*ab*' as shown. To achieve a modulation index of 0.8. the angle θ expressed in degrees should be ________ .

(Round off to 2 decimal places,)

(Modulation index is defined as the ratio of the peak of the fundamental component of V_{ab} to the applied DC value.)





Ans. **51.10 (50.00 to 52.00)**

$$\widehat{V_{01}} = m_a V_S = 0.8 \times 325 = 260 \text{ V}$$

$$\widehat{V_{01}} = \frac{4V_S}{\pi}\sin d = 260$$

$$\frac{4(325)}{\pi}\sin d = 260$$

$$\sin d = \frac{260 \times \pi}{4 \times 325} = 0.628$$

$$d = 38.9$$

$$\therefore \quad \theta = \frac{\pi}{2} - d = 90° - 38.9 = 51.1$$

Q.32 Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

(a) pq and $-pq$
(b) 1 and 1
(c) j and $-j$
(d) 1 and –1

Ans. **(d)**

Characteristic equation of A

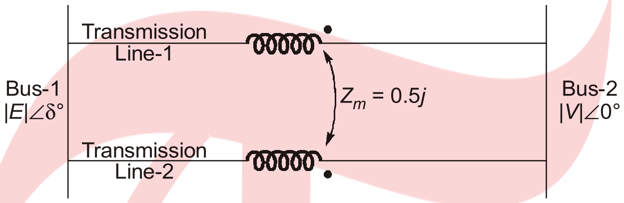
$$|A_{2\times2} - \lambda I| = (-1)^2\lambda^2 + (-1)^1 \text{Tr}(A)\lambda + |A| = 0$$

$$\lambda^2 - (p - p)\lambda + \left(-p^2 - q^2\right) = 0$$

$$\Rightarrow \quad \lambda^2 - 1 = 0$$

$$\Rightarrow \quad \lambda = \pm 1$$

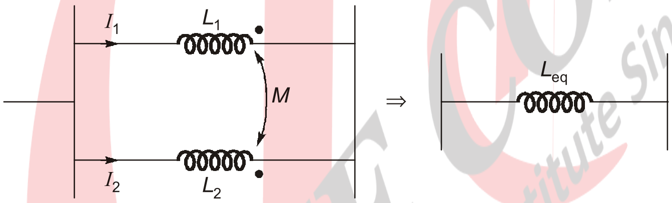
Q.33 In the figure shown, self-impedances of the two transmission lines are 1.5*j* p.u each, and Z_m = 0.5*j* p.u is the mutual impedance. Bus voltages shown in the figure are in p.u. Given that $\delta > 0$, the maximum steady-state real power that can be transferred in p.u from Bus-1 to Bus-2 is



(a) $\dfrac{|E||V|}{2}$ (b) $\dfrac{3|E||V|}{2}$

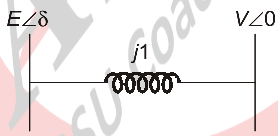
(c) $|E||V|$ (d) $2|E||V|$

Ans. **(c)**



$$L_{eq} = \frac{L_1L_2 - M^2}{(L_1 + L_2 - 2M)}$$

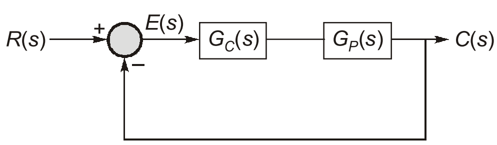
$$X_{eq} = \frac{1.5\times1.5-0.5^2}{1.5+1.5-2\times0.5} = 1 \text{ p.u.}$$



$$P_{max} = \frac{|E||V|}{1}$$

$$P_{max} = |E||V|$$

Q.34 Consider a closed-loop system as shown. $G_p(s) = \dfrac{14.4}{s(1+0.1s)}$ is the plant transfer function and $G_c(s)$ = 1 is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is _______rad/s. (Round off to 2 decimal places.)



Ans. **10.90 (10.80 to 11.00)**

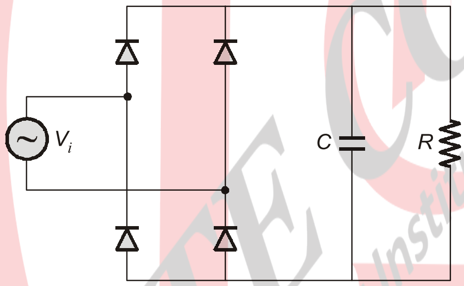
$$q(s) = s^2 + 10s + 144 = 0$$

$$\omega_n = 12;\ \xi = \frac{5}{12}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 12\sqrt{\frac{119}{144}} = 10.90$$

Q.35 In the circuit shown, the input V_i is a sinusoidal AC voltage having an RMS value of 230 V ± 20%. The worst-case peak-inverse voltage seen across any diode is ________V. (Round off to 2 decimal places.)



Ans. **390.32 (385.00 to 395.00)**

$$(V_D)_{peak} \text{ for worst case} = (230 + 20\%) \times \sqrt{2}$$

$$= \left(230 + 230 \times \frac{20}{100}\right) \times \sqrt{2} = 390.32 \text{ V}$$

Q.36 Which one of the following vector functions represents a magnetic field $\vec{B}$?
($\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vectors along x-axis, y-axis and z-axis, respectively)

(a) $10x\hat{X} + 20y\hat{Y} - 30z\hat{Z}$
(b) $10x\hat{X} - 30z\hat{Y} + 20y\hat{Z}$
(c) $10z\hat{X} + 20y\hat{Y} - 30x\hat{Z}$
(d) $10y\hat{X} + 20x\hat{Y} - 10z\hat{Z}$

Ans. **(a)**

If $\vec{B}$ is magnetic flux density then $\vec{\nabla} \cdot \vec{B} = 0$

$$\vec{\nabla} \cdot \vec{B} = \frac{\partial Bx}{\partial x} + \frac{\partial By}{\partial y} + \frac{\partial Bz}{\partial z}$$

$$\frac{\partial}{\partial x}(10x)+\frac{\partial}{\partial y}(20y)+\frac{\partial}{\partial z}(-30z) = \vec{\nabla}\cdot\vec{B}$$

$$\vec{\nabla}\cdot\vec{B} = 10 + 20 - 30 = 0$$

Q.37 Consider the table given:

Constructional feature	Machine type	Mitigation
(P) Damper bars	(S) Induction motor	(X) Hunting
(Q) Skewed rotor slots	(T) Transformer	(Y) Magnetic locking
(R) Compensating winding	(U) Synchronous machine	(Z) Armature reaction
	(V) DC machine	

The correct combination that relates the constructional feature, machine type and mitigation is

(a) P-U-X, Q-V-Y, R-T-Z
(b) P-V-X, Q-U-Z, R-T-Y
(c) P-U-X, Q-S-Y, R-V-Z
(d) P-T-Y, Q-V-Z, R-S-X

Ans. (c)

Q.38 Let A be a 10 × 10 matrix such that A^5 is null matrix, and let I be the 10 × 10 identity matrix. The determinant of $A + I$ is ______ .

Ans. (1)

Given: $A^5 = 0$

$Ax = \lambda x$

$\Rightarrow$ $A^5 x = \lambda^5 x$ $(\because x \neq 0)$

$\Rightarrow$ $\lambda^5 = 0$

$\Rightarrow$ $\lambda = 0$

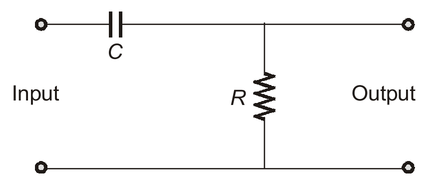
Eigen values of $A + I$ given $\lambda + 1$

$\because$ Eigen values of $I_A = 1$

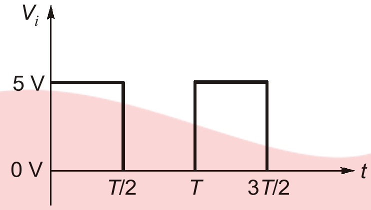
Hence $|A+I|$ = Product of eigen values = 1 × 1 × 1 × ... 10 times

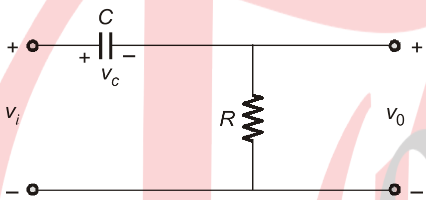
= 1

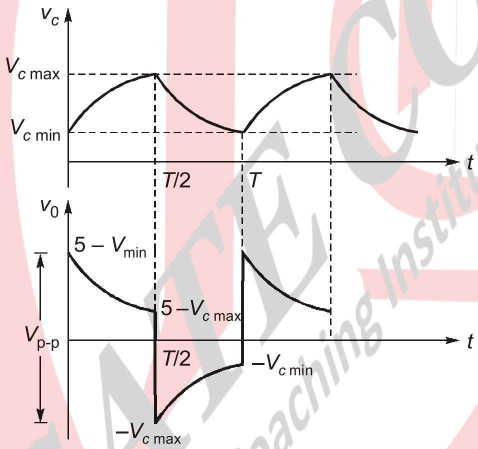
Q.39 A 100 Hz square wave, switching between 0 V and 5 V, is applied to a CR high-pass filter circuit as shown, The output voltage waveform across the resistor is 6.2 V peak-to-peak, If the resistance R is 820 Ω. then the value C is _____,uF. (Round off to 2 decimal places.)



Ans. 12.46 (12.00 to 13.00)







$$v_0 = v_i - v_c$$

For 1st half cycle, $v_0 = 5 - v_c$

For 2nd half cycle, $v_0 = -v_c$

$$V_{p-p} = (5 - V_{c\,min}) - (-V_{c\,max})$$

$$6.2 = 5 + V_{c\,max} - V_{c\,min}$$

$\Rightarrow$ $V_{c\,max} - V_{c\,min} = 1.2$ ...(α)

For first half cycle i.e. $0 < t < \frac{T}{2}$

$$v_c(0^+) = v_c(0) = v_c(0^-) = v_{c\,min}$$

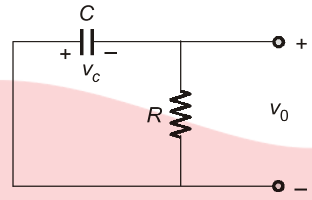
$$v_c(\infty) = 5\text{ V}$$

$\therefore$ $$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-t/\tau}$$

$$v_c(t) = 5 + \left[V_{cmin} - 5\right]e^{-t/2\tau} = V_{cmax}$$

$\Rightarrow$ $$V_{c\,max} = 5\left[1 - e^{-T/2\tau}\right] + V_{cmin}e^{-T/2\tau}$$ ...(i)

For $\frac{T}{2} < t < T$



$$v_c(t) = v_c\left(\frac{T}{2}\right)e^{-t(t-T/2)\tau}$$

$\therefore$ $$v_c(t) = V_{cmax}e^{-(t-T/2)\tau}$$

at $t = T$, $$v_c = V_{c\,min}$$

$\Rightarrow$ $$V_{c\,min} = V_{cmax}e^{-T/2\tau} \qquad \text{...(ii)}$$

As $V_{c\,max} - V_{c\,min} = 1.2$ [From (α)]

$\therefore$ $V_{cmax} - V_{c\,max}\, e^{-T/2t} = 1.2$

$$V_{c\,max} = \frac{1.2}{1-e^{-T/2\tau}} \qquad \text{...(iii)}$$

$\Rightarrow$ $$V_{c\,max} = \frac{1.2}{1-e^{-T/2\tau}} = 5\left[1-e^{-T/2\tau}\right]+V_{cmin}e^{-2\tau}$$

From (ii),

$$V_{c\,max} = 5\left[1-e^{-T/2\tau}\right]+\left(V_{cmax}e^{-T/2\tau}\right)e^{-T/2\tau}$$

$$V_{c\,max} = 5\left[1-e^{-T/2\tau}\right]+V_{cmax}e^{-T/\tau}$$

$\Rightarrow$ $$V_{cmax}\left[1-e^{-T/\tau}\right] = 5\left[1-e^{-T/2\tau}\right]$$

$$V_{c\,max} = \frac{5\left[1-e^{-T/2\tau}\right]}{\left[1+e^{-T/2\tau}\right]\left[1-e^{-T/2\tau}\right]}$$

Using equation (iii),

$$\frac{1.2}{1-e^{-T/2\tau}} = \frac{5}{1+e^{-T/2\tau}}$$

$\Rightarrow$ $1.2 + 1.2\ e^{-T/2\tau} = 5 - 5\ e^{-T/2t}$

$\Rightarrow$ $6.2\ e^{-T/2\tau} = 3.8$

$$e^{-T/2\tau} = \frac{3.8}{6.2} = 0.6129$$

$$\frac{T}{2\tau} = 0.4895$$

as $\quad T = \frac{1}{f} = \frac{1}{100}$ sec

and $\quad \tau = RC = 820\ C$

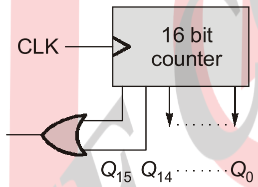
$\Rightarrow \quad \frac{1}{(100)(2)(820)C} = 0.4895$

$\therefore \quad C = 12.46\ \mu F$

Q.40 A 16-bit synchronous binary up-counter is clocked with a frequency f_{CLK} . The two most significant bits are OR-ed together to form an output Y. Measurements show that Y is periodic, and the duration for which Y remains high in each period is 24 ms. The clock frequency f_{CLK} is ________ MHz.

(Round off to 2 decimal places.)

Ans. 2.05 (2.00 to 2.10)



$y = 1$ for 24 m sec

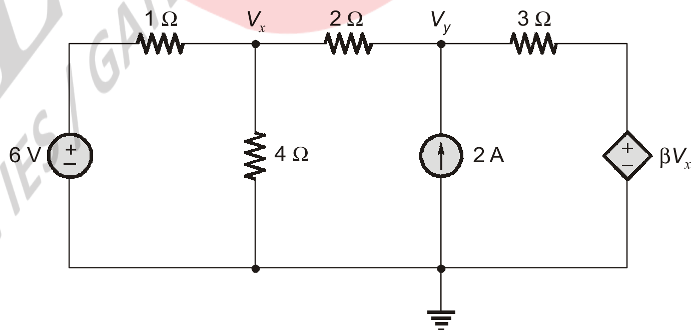
		Q_{15}	Q_{14}	......	Q_0
$y = 1$	Starts at	0100	0000	0000	0000
$y = 1$	Ends at	1111	1111	1111	1111

Total number of states for which $y = 1$ is $2^{16} - 2^{14}$

Time is $(2^{16} - 2^{14})T = 24$ msec

$$f = \frac{1}{T} = 2.05 \text{ MHz}$$

Q.41 In the given circuit, for voltage V_y to be zero, the value of β should be _______ . (Round off to 2 decimal places).



Ans. (–3.25) (–3.30 to –3.20)

$$\frac{V_x - 6}{1} + \frac{V_x}{4} + \frac{V_x - V_y}{2} = 0$$

$$4V_x - 24 + V_x + 2V_x - 2V_y = 0$$

$$7V_x - 2V_y = 24 \quad \text{...(i)}$$

If $V_y = 0$

$$\Rightarrow \quad 7V_x = 24$$

$$\Rightarrow \quad V_x = \frac{24}{7}$$

$$\frac{V_y - V_x}{2} + \frac{V_y - \beta V_x}{3} = 2$$

$$3V_y - 3V_x + 2V_y - 2\beta V_x = 12$$

$$5V_y - (3 + 2\beta)V_x = 12 \quad \text{...(ii)}$$

$$V_y = 0$$

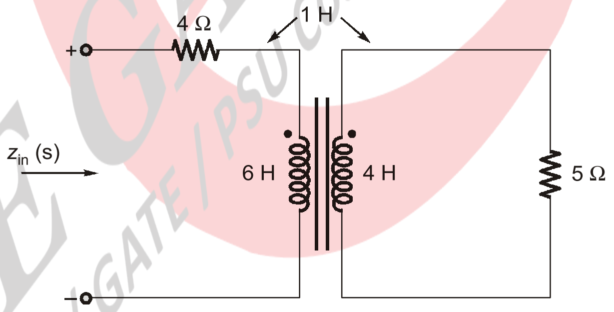
$$-(3 + 2\beta)\frac{24}{7} = 12$$

$$(3 + 2\beta) = \frac{-7}{2}$$

$$2\beta = \frac{-7}{2} - 3 = \frac{-7 - 6}{2} = \frac{-13}{2}$$

$$\beta = \frac{-13}{4} = -3.25$$

Q.42 The input impedance $Z_{in}(s)$, for the network shown in



(a) $7s + 4$

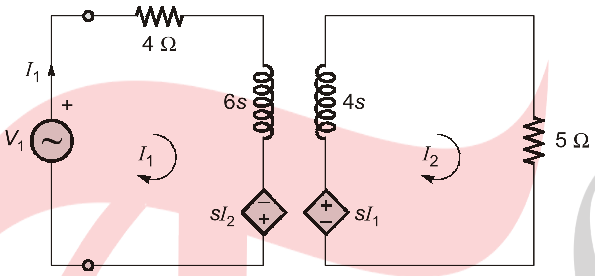
(b) $\dfrac{23s^2 + 46s + 20}{4s + 5}$

(c) $\dfrac{25s^2 + 46s + 20}{4s + 5}$

(d) $6s + 4$

Ans. (b)

Circuit in s-domain,



$$-sI_1 + (4s + 5)I_2 = 0$$

$$\Rightarrow \quad I_2 = \frac{s}{4s+5} I_1$$

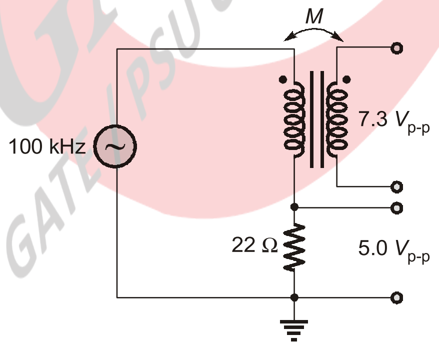
$$V_1 = (4+6s)I_1 - \frac{s^2}{4s+5} I_1$$

$$\frac{V_1}{I_1} = \frac{(4+6s)(4s+5) - s^2}{4s+5}$$

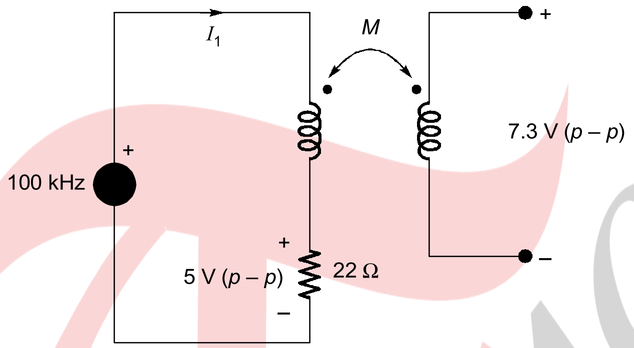
$$= \frac{24s^2 + 30s + 16s + 20 - s^2}{4s+5}$$

$$Z_{in} = \frac{23s^2 + 46s + 20}{4s+5}$$

Q.43 An air-core radio-frequency transformer as shown has a primary winding and a secondary winding. The mutual inductance M between the windings of the transformer is ______ μH, (Round off to 2 decimal places.)



Ans. 51.10 (50.00 to 52.00)



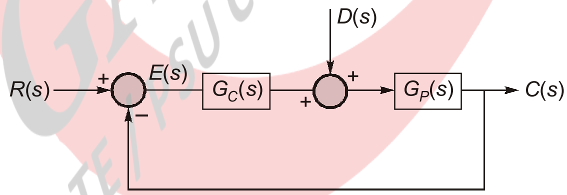
$$I_1 = \frac{5}{22}(p-p)$$

$$V_0 = j\omega M I_1 = 7.3 = (2\pi \times 10 \times 10^3) \times M \times \left(\frac{5}{22}\right)$$

$$M = 51.10\ \mu\text{H}$$

Q.44 In the given figure, plant $G_p(s) = \dfrac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s) = K\left(\dfrac{1+T_1 s}{1+T_2 s}\right)$. The external disturbance input is $D(s)$. It is desired that when the disturbance is a unit step, the steady-steady error should not exceed 0.1 unit. The minimum value of K is ________ .

(Round off to 2 decimal places)



Ans. –10.45 (–11.00 to –10.00)

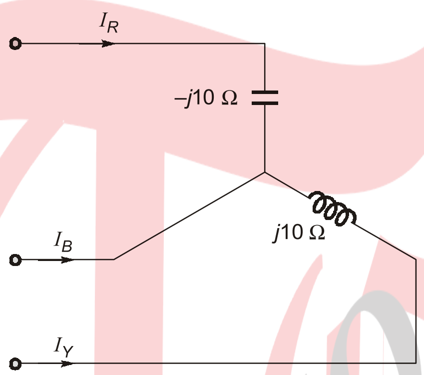
$$e_{ss} = \lim_{s\to 0}\left[\frac{sR}{1+G_C G_P} - \frac{sDG_P}{1+G_C G_P}\right]$$

$$R(s) = 0;\ D(s) = \frac{1}{s}$$

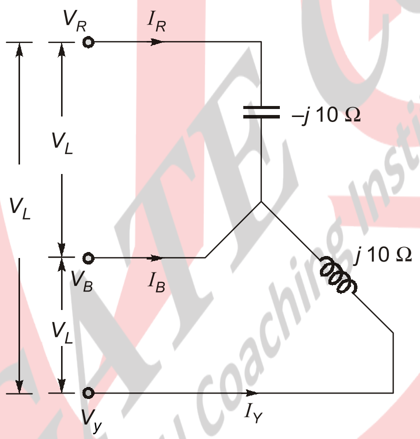
$\therefore$ $$e_{ss} = \frac{2.2}{1+2.2K} = 0.1$$

$\therefore$ $$K_{min} = -10.45$$

Q.45 A three-phase balanced voltage is applied to the load shown. The phase sequence is RYB. The ratio $\frac{|I_B|}{|I_R|}$ is ________ .



Ans. (1)



$$I_R = \frac{V_{RB}}{-j10} = \frac{V_L\angle -60^\circ}{-j10} = \frac{V_L}{10}\angle 30^\circ$$

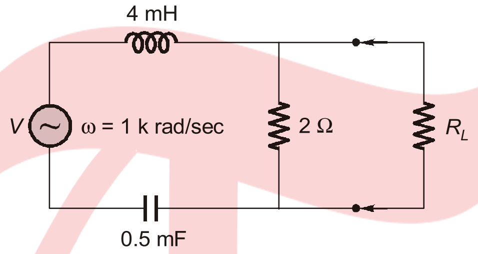
$$I_Y = \frac{V_{YB}}{j10} = \frac{V_L\angle -120^\circ}{-j10} = \frac{V_L}{10}\angle 150^\circ$$

and $$I_B = -(I_R + I_Y) = \frac{V_L}{10}\angle -90^\circ$$

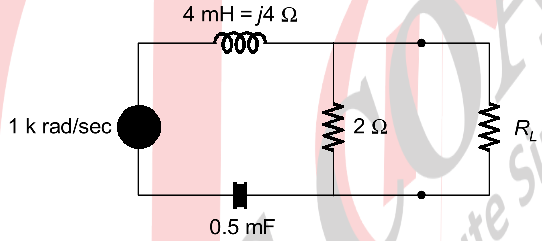
$\therefore$ $$\left|\frac{I_B}{I_R}\right| = 1$$

Q.46 In the given circuit, for maximum power to be delivered to R_L, its value should be ________ Ω.

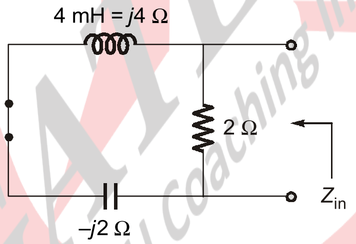
(Round off to 2 decimal places)



Ans. 1.41 (1.40 to 1.42)



$$= \frac{-j}{\omega C} = \frac{-j}{1000 \times 0.5 \times 10^{-3}} = -j2\,\Omega$$



$$Z_{in} = 2 \,||\, j2 = \frac{j4}{2+j2} = \frac{j2}{1+j1}$$

For maximum power transfer,

$$R_L = |Z_{TH}| = \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414\,\Omega$$

Q.47 In a single-phase transformer, the total iron loss is 2500 W at nominal voltage of 440 V and frequency 50 Hz. The total iron loss is 850 W at 220 V and 25 Hz. Then, at nominal voltage and frequency, the hysteresis loss and eddy current loss respectively are

(a) 250 W and 600 W
(b) 900 W and 1600 W
(c) 1600 W and 900 W
(d) 600 W and 250 W

Ans. (b)

$$W_{i1} = 2500 \text{ W at } 440 \text{ V, } 50 \text{ Hz}$$
$$W_{i2} = 850 \text{ W at } 220 \text{ V, } 25 \text{ Hz}$$
$$W_{i3} = R_{e3} + P_{h3} \text{ at } 440 \text{ V, } 50 \text{ Hz}$$
$$R_{e3} = ?, \quad P_{h3} = ?, \quad \frac{V}{f} = \text{constant}$$

$$\Rightarrow \quad 2500 = Af + Bf^2 \qquad \left\{\frac{400}{50} = \frac{220}{25} = \text{Constant}\right\}$$

or, $$\frac{2500}{f} = A + Bf$$

or, $$\frac{2500}{50} = A + B(50) \qquad \text{...(i)}$$

and $$\frac{850}{25} = A + B\ (25) \qquad \text{...(ii)}$$

Solving (i) and (ii), we get

$$25B = \frac{2500}{50} - \frac{850}{25} = \frac{2500-1700}{50}$$
$$= \frac{800}{50} = 16$$
$$B = \frac{16}{25}$$

and from (i), $$A = 50 - \frac{16}{25} \times 50 = 50 - 32 = 18$$

So, at 50 Hz

$$P_h = Af = 18 \times 50 = 900 \text{ W}$$
$$P_e = Bf^2 = \left(\frac{16}{25}\right) \times (50)^2 = 1600 \text{ W}$$

Q.48 The causal signal with z-transform $z^2(z-a)$ is ($u[n]$ is the unit step signal)

(a) $n^{-1} a^n u[n]$ (b) $(n+1)a^n u[n]$

(c) $n^2 a^n u[n]$ (d) $a^{2n} u[n]$

Ans. (b)

As we know,

$$n.a^n u(n) \rightleftharpoons \frac{az}{(z-a)^2}$$
$$\frac{1}{a}.n.a^n u(n) \rightleftharpoons \frac{z}{(z-a)^2}$$
$$f(n) = n.a^{n-1}u(n) \rightleftharpoons \frac{z}{(z-a)^2} = F(z)$$

Time-shifting property,

$$f(n) \rightleftharpoons F(z)$$
$$f(n+1) \rightleftharpoons z.F(z)$$

$$x(n) = (n+1)a^n u(n+1) \rightleftharpoons \frac{z^2}{(z-a)^2} = X(z)$$

Thus, $x(n) = (n+1)a^n u(n+1) = (n+1)a^n.u(n)$

$[\because (n+1)u(n+1) = (n+1)u(n)]$

Q.49 Let $f(x)$ be a real-valued function such that $f'(x_0) = 0$ for some $x_0 \in (0, 1)$ and $f''(x) > 0$ for all $x \in (0, 1)$. Then $f(x)$ has

(a) two distinct local minima in (0, 1)
(b) exactly one local minimum in (0, 1)
(c) one local maximum in (0, 1)
(d) no local minimum in (0, 1)

Ans. (b)

$x_0 \in (0, 1)$, where $f'(x) = 0$ is stationary point

and $f''(x) > 0$ $\quad \forall x \in (0,1)$

So $f'(x_0) = 0$

and $f''(0) > 0$, where $x_0 \in (0, 1)$

Hence, $f(x)$ has exactly one local minima in (0, 1)

Q.50 Suppose I_A, I_B and I_C are a set of unbalanced current phasors in a three-phase system. The phase-B zero sequence current $I_{B0} = 0.1\angle 0°$ p.u. If phase-A current $I_A = 1.1\angle 0°$ p.u. and phase-C current $I_c = (1\angle 120° + 0.1)$ p.u., then I_B in p.u. is

(a) $1.1\angle 240° - 0.1\angle 0°$
(b) $1\angle 240° - 0.1\angle 0°$
(c) $1\angle -120° + 0.1\angle 0°$
(d) $1.1\angle -120° + 0.1\angle 0°$

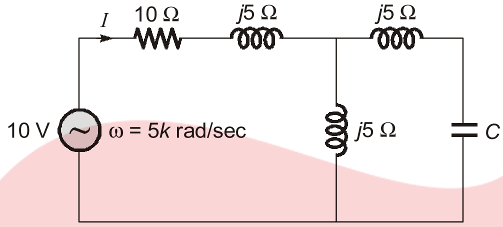
Ans. (c)

$$I_{B0} = \frac{1}{3}(I_A + I_B + I_C)$$

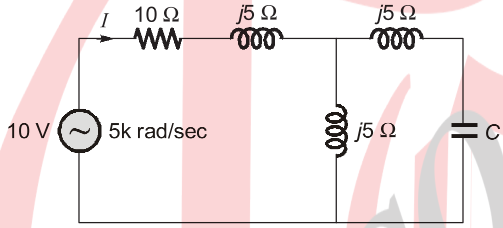
$$0.1 = \frac{1}{3}(1.1\angle 0 + I_B + 1\angle 120° + 0.1)$$

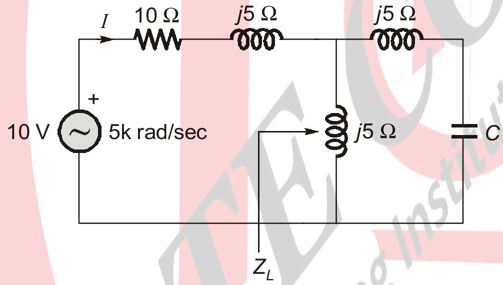
$$I_B = 0.3 - 1.1\angle 0 - 0.1 - 1\angle 120°$$
$$I_B = -0.9 - 1\angle 120°$$
$$I_B = 0.1 + 1\angle 240°$$
$$I_B = 1\angle -120° + 0.1\angle 0°$$

Q.51 In the given circuit, the value of capacitor C that makes current $I = 0$ is ______ μF.



Ans. 20.00 (19.90 to 20.10)





$$Z_L = (j5) \parallel (j5 - jX_c)$$

$$\frac{(j5)(j5 - jX_c)}{j5 + j5 - jX_c} = \infty$$

$$j5 + j5 - jX_c = 0$$

$$\Rightarrow \quad jX_c = j10$$

$$\Rightarrow \quad X_c = 10\ \Omega$$

$$X_c = \frac{1}{\omega C} = 10$$

$$\Rightarrow \quad C = \frac{1}{10 \times 5 \times 10^3}$$

$$C = \frac{1}{5 \times 10^4} \times \frac{10^2}{10^2} = \frac{10^2}{5 \times 10^6} = 20\ \mu\text{F}$$

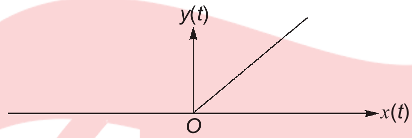
Q.52 If the input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \max(0, x(t))$, then the system is

(a) non-linear and time variant
(b) linear and time-variant
(c) linear and time-invariant
(d) non-linear and time-invariant

Ans. (d)

$$y(t) = \max(0, x(t))$$

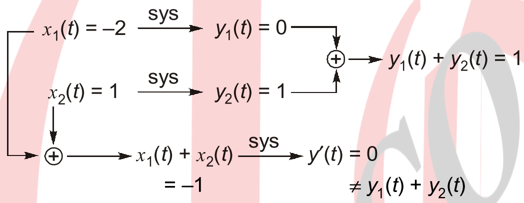
$$= \begin{cases} 0, & x(t) < 0 \\ x(t), & x(t) > 0 \end{cases}$$



Linearity check :

at input $x_1(t) = -2$, output $y_1(t) = 0$

at input $x_2(t) = 1$, output $y_2(t) = 1$



∴ system is non-linear because it violates law of additivity.

Check for time-invariance :

Delayed O/P :

$$y(t - t_o) = \begin{cases} x(t - t_o), & x(t - t_o) > 0 \\ 0 & x(t - t_o) < 0 \end{cases}$$

O/P of system when input is $x(t - t_o) = f(t)$

$$y_1(t) = \begin{cases} f(t), & f(t) > 0 \\ 0, & f(t) < 0 \end{cases} = \begin{cases} x(t - t_o), & x(t - t_o) > 0 \\ 0, & x(t - t_o) < 0 \end{cases}$$

Therefore, system is time-invariant.

Q.53 Two generators have cost functions F_1 and F_2. Their incremental-cost characteristics are

$$\frac{dF_1}{dP_1} = 40 + 0.2P_1 \; ; \quad \frac{dF_2}{dP_2} = 32 + 0.4P_2$$

They need to deliver a combined load of 260 MW. Ignoring the network losses, for economic operation, the generation P_1 and P_2 (in MW) are

(a) $P_1 = 160$, $P_2 = 100$
(b) $P_1 = P_2 = 130$
(c) $P_1 = 120$, $P_2 = 140$
(d) $P_1 = 140$, $P_2 = 120$

Ans. (a)

$$IC_1 = IC_2$$

$$40 + 0.2P_1 = 32 + 0.4P_2$$

$$0.4P_2 - 0.2P_1 = 8 \qquad \text{...(i)}$$

$$P_2 + P_1 = 260 \qquad \text{...(ii)}$$

Solving equation (i) and (ii),

$$P_1 = 160 \text{ MW}; \quad P_2 = 100 \text{ MW}$$

Q.54 Suppose the probability that a coin toss shows "head" is p, where $0 < p < 1$. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is

(a) $\frac{(1-p)}{p}$ (b) $\frac{1}{p}$

(c) $\frac{1}{p^2}$ (d) $\frac{p}{(1-p)}$

Ans. (b)

$P(H) = p$, let x = Number of tosses

x	1	2	3	4	5...
$P(x)$	p	$(1-p)p$	$(1-p)^2 p$		

$$E(x) = \sum x_i P(x) = p + 2(1-p)p + 3(p)(1-p)^2 + ...$$

$$= p\left[1 + 2(1-p) + 3(1-p)^2 + ...\right]$$

$$= p[1-(1-p)]^{-2} = \frac{1}{p}$$

Q.55 Two discrete-time linear time-invariant systems with impulse response $h_1[n] = \delta[n-1] + \delta[n+1]$ and $h_2[n] = \delta[n] + \delta[n-1]$ are connected in cascade, where $\delta[n]$ is the Kronecker delta. The impulse response of the cascaded system is

(a) $\delta[n-2] + \delta[n+1]$ (b) $\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$

(c) $\delta[n-1]\,\delta[n] + \delta[n+1]\,\delta[n-1]$ (d) $\delta[n]\,\delta[n-1] + \delta[n-2]\,\delta[n+1]$

Ans. (b)

$h(n)$ = Resultant impulse response

$= h_1(n) * h_2(n)$

By applying z-transform

$$H(z) = H_1(z).H_2(z)$$

$$= (z + z^{-1})(1 + z^{-1})$$

$$= z + z^{-1} + 1 + z^{-2}$$

By applying inverse ZT,

$$h(n) = \delta(n+1) + \delta(n-1) + \delta(n) + \delta(n-2)$$