GATE 2021

## CIVIL ENGINEERING (Morning)

## GENERAL APTITIDE

Q. 1 - Q. 5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: 1/3).

1. Getting to the top is $\qquad$ than staying on top.
(a) more easy
(b) much easy
(c) easiest
(d) easier

Ans. d
Exp:
When the comparison is between the two things we use the second degree of the adjective. The degree form of easy are : easy $\rightarrow$ easier $\rightarrow$ easiest.
2. The mirror image on the above text about the $x$-axis is
(a) the mirror image of the above text about the x -axis is

(a) $\perp$ ВIVИЭГЕ
(b) $\perp$ ВIVИСГЕ
(c) $\perp$ bIVNCГE
(d) $\perp$ ВІИИСГ

Ans. b

Exp:

3. In a company, $35 \%$ of the employees drink coffee, $40 \%$ of the employees drink tea and $10 \%$ of the employees drink both tea and coffee. What $\%$ of employees drink neither tea nor coffee?
(a) 15
(b) 25
(c) 35
(d) 40

Ans. c

Exp:
Percent of employees drink neither tea nor coffee $=100-25-10-30=35$

$$
T=100 \%
$$


4. $\oplus$ and $\odot$ are two operators on numbers $p$ and $q$ such that
$\mathrm{p} \oplus \mathrm{q}=\frac{\mathrm{p}^{2}+\mathrm{q}^{2}}{\mathrm{pq}} \quad$ and $\quad \mathrm{p} \odot \mathrm{q}=\frac{\mathrm{p}^{2}}{\mathrm{q}} ;$
If $x \oplus y=2 \odot 2$, then $x=$

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(a) $\frac{\mathrm{y}}{2}$
(b) y
(c) $\frac{3 y}{2}$
(d) 2 y

Ans. b
Exp:
Given that, If $x \oplus y=2 \odot 2$,
$\frac{x^{2}+y^{2}}{x y}=\frac{2^{2}}{2}$
$x^{2}+y^{2}=2 x y$
$(x-y)^{2}=0$
$x=y$
5. Four persons $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S are to be seated in a row, all facing the same direction, but not necessarily in the same order. $P$ and $R$ cannot sit adjacent to each other. $S$ should be seated to the right of Q . The number of distinct seating arrangements possible is :
(a) 2
(b) 4
(c) 6
(d) 8

Ans. c
Exp:
Following cases can be
PQRS, PQSR, QPSR, RQSP, QRSP, RQPS

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Q. 6 - Q. 10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: $-2 / 3)$.
6. Statement: Either P marries Q or X marries Y

Among the option below, the logical NEGATION of the above statement is:
(a) P does not marry Q and X marries Y
(b) Neither P marries Q nor X marries Y
(c) X does not marry Y and P marries Q
(d) P marries Q and X marries Y

Ans. b

## Exp:

The statement says only one of these two action will happen, its NEGATION should be a confirmed action.
7. Consider two rectangular sheets, Sheet $M$ and Sheet $N$ of dimensions $6 \mathrm{~cm} \times 4 \mathrm{~cm}$ each.

Folding operation 1: The sheet is folded into half by joining the short edges of the current shape.
Folding operation 2: The sheet is folded into half by joining the long edges of the current shape.
Folding operation 1 is carried out on Sheet M three times.
Folding operation 2 is carried out on Sheet N three times.
The ratio of perimeter of the final folded shape of Sheet N to the final folded shape of Sheet M is
(a) $13: 7$
(b) $3: 2$
(c) $7: 5$
(d) $5: 13$

Ans: a
Exp:

8. Five line segments of equal lengths, PR, PS, QS, QT and RT are used to form a star as shown in the figure below.

The value of $\theta$ is $\qquad$ (degree).

(a) 36
(b) 45
(c) 72
(d) 108

Ans. a


Sum of angle formed at the pentagon $=540^{\circ}$
Each angle of pentagon $=\frac{540}{5}=108^{\circ}$
$\angle x=180-108=72^{\circ}$
Sum of angle of triangle $=180^{\circ}$
$72^{\circ}+72^{\circ}+\theta=180^{\circ}$
$\theta=36^{\circ}$
9. A function, $\lambda$, is defined by
$\lambda(p, q)=\left\{\begin{array}{ll}(p-q)^{2}, & \text { if } p \geq q \\ p+q, & \text { if } p<q\end{array}\right\}$

The value of the expression $\frac{\lambda(-(-3+2),(-2+3))}{(-(2+1))}$ is
(a) -1
(b) 0
(c) $\frac{16}{3}$
(d) 16

Ans. b

Exp:

$$
\frac{\lambda(-(-3+2),(-2+3))}{(-(2+1))}=\lambda \frac{(1,1)}{1}=\lambda(1,1)
$$

So, $1^{\text {st }}$ definition will be applicable as $\mathrm{p}=\mathrm{q}$.

Hence, $\lambda(1,1)=(1-1)^{2}=0$
10. Human have the ability to construct worlds entirely in their minds, which don't exist in the physical world. So, far as we know, no other species possesses the ability. This skill is so important that we have different worlds to refer to its different flavours, such as imagination invention and innovation.

Based on the above passage, which one of the following is TRUE?
(a) No spices possess the ability to construct worlds in their minds.
(b) The terms imagination, invention and innovation refer to unrelated skill.
(c) We do not know of any species other than humans who possess the ability to construct mental worlds.
(d) Imagination, invention and innovation are unrelated to the ability to construct mental worlds.

Ans. c

## CIVIL ENGINEERING (TECHNICAL)

Q. 1 -Q. 16 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: 1/3).

1. The rank of matrix $\left[\begin{array}{llll}1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9\end{array}\right]$ is
(a) 1
(b) 2
(c) 3
(d) 4

Ans. b
Exp:
Using $\mathrm{R}_{2} \rightarrow \mathrm{R}_{2}-3 \mathrm{R}_{1}, \mathrm{R}_{3} \rightarrow \mathrm{R}_{3}-5 \mathrm{R} 1, \mathrm{R}_{4} \rightarrow \mathrm{R}_{4}-7 \mathrm{R}_{1}$
$A=\left[\begin{array}{cccc}1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & -4 & -8 & -8 \\ 0 & -6 & -12 & -12\end{array}\right]$
Using $\mathrm{R}_{3} \rightarrow \mathrm{R}_{3}-2 \mathrm{R}_{2}, \mathrm{R}_{4} \rightarrow \mathrm{R}_{4}-3 \mathrm{R}_{2}$
$A=\left[\begin{array}{cccc}1 & 2 & 2 & 3 \\ 0 & -2 & -4 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$
So, Rank of matrix $=$ No. of non-zero rows $=2$.
2. If $P=\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$ and $Q=\left[\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right]$ then $Q^{T} P^{T}$ is
(a) $\left[\begin{array}{ll}1 & 2 \\ 3 & 4\end{array}\right]$
(b) $\left[\begin{array}{ll}1 & 3 \\ 2 & 4\end{array}\right]$

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(c) $\left[\begin{array}{ll}2 & 1 \\ 4 & 3\end{array}\right]$
(d) $\left[\begin{array}{ll}2 & 4 \\ 1 & 3\end{array}\right]$

Ans. d
Exp:

$$
\begin{aligned}
& P Q=\left[\begin{array}{ll}
1 & 3 \\
2 & 4
\end{array}\right]\left[\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right]=\left[\begin{array}{ll}
2 & 4 \\
1 & 3
\end{array}\right] \\
& (P Q)^{T}=\left[\begin{array}{ll}
2 & 4 \\
1 & 3
\end{array}\right]
\end{aligned}
$$

Now using Reversal law,

$$
Q^{T} P^{T}=(P Q)^{T}=\left[\begin{array}{ll}
2 & 4 \\
1 & 3
\end{array}\right]
$$

3. The shape of the cumulative distribution function of Gaussian distribution is
(a) horizontal line
(b) Straight line at 45 degree angle
(c) Bell-shaped
(d) S-shaped

Ans. d
4. A propped cantilever beam EF is subjected to a unit moving load as shown in the figure (not to scale). The sign convention for positive shear force at the left and right sides of any section is also shown.


## Sign convention for positive shear force

The correct qualitative nature of the influence line diagram for shear force at G is
(a)

(b)

(d)


Ans. b

Exp.


As per Muller Breslau principle ILD for stress function (shear $-\mathrm{V}_{\mathrm{G}}$ ) will be a combination of curves ( $3^{\circ}$ curves).

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5. Gypsum is typically added in cement to
(a) prevent quick setting
(b) enhance hardening
(c) increase workability
(d) decrease heat of hydration

Ans. a
6. The direct and indirect costs estimated by a contractor for bidding a project is Rs. 1,60,000 and Rs. 20,000 respectively. If the mark up applied is $10 \%$ of the bid price, the quoted price (in Rs.) of the contractor is
(a) $2,00,000$
(b) $1,98,000$
(c) $1,96,000$
(d) $1,82,000$

Ans. a

## Exp:

Total cost estimated $=$ Rs. $160000+$ Rs. $20000=$ Rs. 180,000
Mark up $=10 \%$ (above the actual cost incurred)
$\therefore \quad$ Mark up cost $=$ Rs. 18000
$\therefore \quad$ Quoted price $=$ Rs. $180000+$ Rs. $18000=$ Rs. 198000
7. In an Oedometer apparatus, a specimen of fully saturated clay has been consolidated under a vertical pressure of $50 \mathrm{kN} / \mathrm{m}^{2}$ and is presently at equilibrium. The effective stress and pore water pressure immediately on increasing the vertical stress to $150 \mathrm{kN} / \mathrm{m}^{2}$, respectively are
(a) $150 \mathrm{kN} / \mathrm{m}^{2}$ and 0
(b) $100 \mathrm{kN} / \mathrm{m}^{2}$ and $50 \mathrm{kN} / \mathrm{m}^{2}$
(c) $50 \mathrm{kN} / \mathrm{m}^{2}$ and $100 \mathrm{kN} / \mathrm{m}^{2}$
(d) 0 and $150 \mathrm{kN} / \mathrm{m}^{2}$

Ans. c

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## Exp:

Stress is increased suddenly, hence entire change will be taken by water
$\Delta \bar{\sigma}=\Delta \mathrm{U}=100 \mathrm{kPa}$
There will be no change in effective stress.

$$
\therefore \quad \bar{\sigma}=50 \mathrm{kPa}
$$

8. A partially-saturated soil sample has natural moisture content of $25 \%$ and bulk unit weight of 18.5 $\mathrm{kN} / \mathrm{m}^{3}$. The specific gravity of soil solids is 2.65 and unit weight of water is $9.81 \mathrm{kN} / \mathrm{m}^{3}$. The unit weight of the soil sample on full saturation is
(a) $21.12 \mathrm{kN} / \mathrm{m}^{3}$
(b) $19.03 \mathrm{kN} / \mathrm{m}^{3}$
(c) $20.12 \mathrm{kN} / \mathrm{m}^{3}$
(d) $18.50 \mathrm{kN} / \mathrm{m}^{3}$

Ans. b

Exp:

$$
\begin{aligned}
& \mathrm{w}=0.25, \gamma_{\mathrm{t}}=18.5 \mathrm{kN} / \mathrm{m}^{3} \\
& \mathrm{G}_{\mathrm{S}}=2.65, \gamma_{\mathrm{w}}=9.81 \\
& \gamma_{t}=\frac{G_{s} \gamma_{w}(1+w)}{1+e} \\
& \Rightarrow \quad e=\frac{2.65 \times 9.81 \times 1.25}{18.5}-1=0.756 \\
& \text { At full saturation, } \quad \mathrm{S}=1
\end{aligned}
$$

$$
\Rightarrow \quad \gamma_{s a t}=\frac{\left(G_{s}+e\right) \gamma_{w}}{1+e}
$$

$$
\gamma_{\text {sat }}=\frac{(2.65+0.756) \times 9.81}{1.756}
$$

$$
=19.03 \mathrm{kN} / \mathrm{m}^{3}
$$

9. If water is flowing at the same depth in most hydraulically efficient triangular and rectangular channel sections then the ratio of hydraulic radius of triangular section to that of rectangular section is
(a) $\frac{1}{\sqrt{2}}$
(b) $\sqrt{2}$
(c) 1
(d) 2

Ans. a
Exp:
Efficient channel section

$A=y^{2}$
$P=2 \sqrt{2} y$
$R_{I}=\frac{y}{2 \sqrt{2}}$
$R_{I I}=\frac{y}{2}$
$\frac{R_{I}}{R_{I I}}=\frac{1}{\sqrt{2}}$
10. 'Kinematic viscosity' is dimensionally represented as
(a) $\frac{M}{L T}$
(b) $\frac{M}{L^{2} T}$
(c) $\frac{T^{2}}{L}$
(d) $\frac{L^{2}}{T}$

Ans. d
11. Which one of the following statements is correct?
(a) Pyrolysis is an endothermic process, which takes place in the absence of oxygen
(b) Pyrolysis is an exothermic process, which takes place in the absence of oxygen
(c) Combustion is an endothermic process, which takes place in the abundance of oxygen.
(d) Combustion is an exothermic process, which takes place in the absence of oxygen

Ans. a
Exp:
Pyrolysis is an endothermic process as there is a substantial heat input required to raise the biomass to the reaction temperature.
12. Which one of the following is correct?
(a) The partially treated effluent from a food processing industry, containing high concentration of biodegradable organism is being discharge into a flowing river at a point P . If the rate of degradation of the organics is higher than the rate of aeration, then dissolved oxygen of the river water will be lowest at point $P$.
(b) The most important type of species involved in the degradation of organic matter in the case of activated sludge process based wastewater treatment is chemoheterotrophs.
(c) For an effluent sample of a sewage treatment plant, the ratio $\mathrm{BOD}_{5-\text { day }-20^{\circ} \mathrm{C}}$ upon ultimate BOD is more than 1 .
(d) A young lake characterised by low nutrient content and low plant productivity is called eutrophic lake.

Ans. b
Chemoheterotrophs are the organisms which generate energy with the help of chemical reactions by consuming organic matter during their metabolism. These are the species which help to degrade the organic matter in ASP.
13. The liquid forms of particulate air pollutants are
(a) dust and mist
(b) mist and spray
(c) smoke and spray
(d) fly ash and fumes

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Ans. b

## Exp:

The liquid forms of particulate air pollutants are mist and spray.
Note: Mist is a cloud made of very small drops of water in the air just above the ground which reduces the visibility.
14. The shape of the most commonly designed highway vertical curve is
(a) circular (single radius)
(b) circular (multiple radii)
(c) parabolic
(d) spiral

Ans. c
15. A highway designed for 80 kmph speed has a horizontal curve section with radius 250 m . If the design lateral friction is assumed to develop fully, the required superelevation is
(a) 0.02
(b) 0.05
(c) 0.07
(d) 0.09

Ans. b

## Exp:

$\mathrm{V}=80 \mathrm{kmph}, \mathrm{R}=250 \mathrm{~m}$
$e+f=\frac{V^{2}}{127 R}$
$e+0.15=\frac{80^{2}}{127 \times 250}$
$e=0.051$

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16. Which of the following is NOT a correct statement?
(a) The first reading from a level station is a 'Fore Sight'.
(b) Basic principle of surveying is to work from whole to parts
(c) Contours of different elevations may intersect each other in case of an overhanging cliff.
(d) Planimeter is used for measuring 'area'

Ans. a
Exp:
First reading from level station is called BS.

## Q. 17 Multiple Select Question (MSQ), carry ONE marks (no negative marks).

17. Which of the following is/are correct statement(s)?
(a) Back bearing of a line is equal to Fore bearing $\pm 180^{\circ}$.
(b) If the whole circle bearing of a line is $270^{\circ}$, its reduced bearing is $90^{\circ} \mathrm{NW}$.
(c) The boundary of water of a calm water pond will represent contour line
(d) In the case of fixed hair stadia tachometry, the staff intercept will be larger, when the staff is held nearer to the observation point.

Ans. a, b, c
Exp:
(a)

(b)

(c)


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## Q. 18 - Q. 25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

18. Consider the limit:
$\lim _{x \rightarrow 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$
The limit (correct up to one decimal place) is $\qquad$ .

Ans. 0.5
Exp:
$\lim _{x \rightarrow 1}\left[\frac{1}{\ln x}-\frac{1}{x-1}\right]=\lim _{x \rightarrow 1}\left[\frac{(x-1)-\ln x}{\ln x(x-1)}\right]\left(\frac{0}{0}\right.$ form $)$
So, using L-Hospital's rule twice
$\lim _{x \rightarrow 1}=\left[\frac{1-\frac{1}{x}}{\log x+(x-1)\left(\frac{1}{x}\right)}\right]=0.5$
19. The volume determined from $\iiint_{V} 8 x y z d V$ for $V=[2,3] \times[1,2] \times[0,1]$ will be (in integer)

Ans. 15
Exp:
Given, $V=[2,3] \times[1,2] \times[0,1]$, i.e.,
$2<x<3,1<y<2,0<z<1$
So, $\quad \mathrm{I}=\iiint_{V} 8 x y z d V=8 \iiint_{V} x y z d x d y d z$
$=8 \int_{x=2}^{3} x d x \times \int_{y=1}^{2} y d y \times \int_{z=0}^{1} z d z$
$=8\left[\frac{x^{2}}{2}\right]_{2}^{3}\left[\frac{y^{2}}{2}\right]_{1}^{2}\left[\frac{z^{2}}{2}\right]_{0}^{1}$

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$=(9-4)(4-1)(1-0)$
$=5 \times 3 \times 1=15$
20. The state of stress in a deformable body is shown in the figure. Consider transformation of the stress from the $x-y$ coordinates system to the $x-y$ coordinates system. The angle $\theta$, locating the $X$-axis, is assumed to be positive when measured from the x -axis is counter-clockwise direction.


The absolute magnitude of the shear stress components $\sigma_{x y}$ (in MPa , round off to one decimal place) in $x-y$ coordinate system is $\qquad$ .

Ans. 95.0 - 97.0
Exp:


$$
\sigma_{x}^{\prime}=\frac{\sigma_{x}+\sigma_{y}}{2}+\left(\frac{\sigma_{x}-\sigma_{y}}{2}\right) \cos 2 \theta+\tau_{x y} \sin 2 \theta
$$

Here, $\theta=60^{\circ}$

$$
\sigma_{x}=40 \mathrm{MPa}, \sigma_{y}=35.6, \sigma_{x}^{\prime}=120, \tau_{x^{\prime} y^{\prime}}=-50
$$

Substituting the values in above equation, we get
$\tau_{\text {xy }}=96.186 \mathrm{MPa} \simeq 96.2 \mathrm{MPa}$
21. The equation of deformation is derived to be $y=x^{2}-x \mathrm{~L}$ for a beam shown in the figure.


The curvature of the beam at the mid-span (in units, in integer) will be $\qquad$
Ans. 2
Exp:


Given, $\mathrm{y}=\mathrm{x}^{2}-x l$
Curvature at mid section is $\frac{1}{R}=\frac{d^{2} y}{d x^{2}}$
$\frac{d y}{d x}=2 x-l$
$\frac{d^{2} y}{d x^{2}}=2$
22. A truss EFGH is shown in the figure, in which all the members have the same axial rigidity R. In the figure, P is the magnitude of external horizontal forces acting at joints F and G .

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If $\mathrm{R}=500 \times 10^{3} \mathrm{kN}, \mathrm{P}=150 \mathrm{kN}$ and $\mathrm{L}=3 \mathrm{~m}$, the magnitude of the horizontal displacement of joint G (in mm , round off to one decimal place) is $\qquad$ -.

Ans. 0.9
Exp:


Note: No need to calculate ' $k$ ' force in all members because ' $P$ ' force is zero for all members except FG.

By unit load method,

1. $\Delta_{H G}=\sum_{i=1}^{n} \frac{P_{i} k_{i} L_{i}}{A_{i} E_{i}}$
$\Delta_{\mathrm{HG}}=$ horizontal deflection at joint G

$$
\therefore \quad \Delta_{H G}=\underbrace{\frac{P \times 1 \times L}{A E}}_{F G-\text { member }}+{ }_{\text {(Forallother members) }} \text { 'Q' }
$$

$$
\Delta_{H G}=\frac{P L}{A E}=\left(\frac{150 \times 3}{500 \times 10^{3}} \times 10^{3}\right) \mathrm{mm}
$$

$=0.9 \mathrm{~mm}$
23. The cohesion (c), angle of internal friction ( $\phi$ ) and unit weight ( $\gamma$ ) of a soil are $15 \mathrm{kPa}, 20^{\circ}$ and $17.5 \mathrm{kN} / \mathrm{m}^{3}$, respectively. The maximum depth of unsupported excavation in the soil (in m , round off to two decimal places) is $\qquad$ -.

Ans. $4.80-5.00$

## Exp:

Maximum depth of unsupported excavation,
$H=\frac{4 C}{\gamma_{t} \sqrt{k_{a}}}$
$k_{a}=\frac{1-\sin \phi}{1+\sin \phi}=\frac{1-\sin 20^{\circ}}{1+\sin 20^{\circ}}=0.49$
$H=\frac{4 \times 15}{17.5 \sqrt{0.49}}=4.90 \mathrm{~m}$
24. Two reservoirs are connected through a homogeneous and isotropic aquifer having hydraulic conductivity (k) of $25 \mathrm{~m} /$ day and effective porosity $(\eta)$ of 0.3 as shown in the figure (not to scale). Ground water is flowing in the aquifer at the steady state.


If water is reservoir 1 is contaminated then the time (in days, round off to one decimal place) taken by the contaminated water to reach to Reservoir 2 will be $\qquad$
Ans: 2400
Exp:
$\mathrm{k}=25 \mathrm{~m} / \mathrm{d}$

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$\mathrm{n}=0.3$
$i=\frac{20 \mathrm{~m}}{2000 \mathrm{~m}}=0.01$
$\mathrm{V}=\mathrm{ki}$
$V_{s}=\frac{V}{n}$
$t=\frac{l}{V_{s}}$
$\mathrm{t}=2400$ days
25. A signalized intersection operates in two phases. The lost time is 2 seconds per phase. The maximum ratios of approach flow to saturation flow for the two phases are 0.37 and 0.40 . The optimum cycle length using the Webster's method (in seconds, round off to one decimal place) is
$\qquad$ -.

Ans. 60.7-61.1
Exp:
$\mathrm{n}=2, \mathrm{~L}=3 \times 2=6 \mathrm{sec}$
$\mathrm{y}_{1}=0.37, \mathrm{y}_{2}=0.40$
$\mathrm{Y}=\mathrm{y}_{1}+\mathrm{y}_{2}=0.37+0.40=0.77$
$C_{o}=\left(\frac{1.5 L+5}{1-Y}\right)=\frac{1.5 \times 6+5}{1-0.77}$
$=60.87 \mathrm{sec} \approx 60.9 \mathrm{sec}$
Q. 26 - Q. 35 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: $-2 / 3)$.
26. The solution of the second-order differential equation $\frac{d^{2} y}{d x^{2}}+2 \frac{d y}{d x}+y=0$ with boundary conditions $y(0)=1$ and $y(1)=3$ is
(a) $e^{-x}+(3 e-1) x e^{-x}$
(b) $e^{-x}-(3 e-1) x e^{-x}$
(c) $e^{-x}+\left[3 e \sin \left(\frac{\pi x}{2}\right)-1\right] x e^{-x}$
(d) $e^{-x}-\left[3 e \sin \left(\frac{\pi x}{2}\right)-1\right] x e^{-x}$

Ans. a
Exp:
Given equation is,
$\left(D^{2}+2 D+1\right) y=0$
$y(0)=1$
$y(1)=3$
Auxiliary equation in $\mathrm{m}^{2}+2 \mathrm{~m}+1=0$
$\Rightarrow \quad \mathrm{m}=-1,-1$
C.F. $=\left(C_{1}+C_{2} x\right) \mathrm{e}^{-x}$

And $\quad \mathrm{PI}=0$
$\mathrm{C}_{1}$ solution is
uy $=\mathrm{CF}+\mathrm{PI}$
$y=\left(C_{1}+C_{2} x\right) e^{-x}$
Using $\mathrm{y}(0)=1, \quad \mathrm{C}_{1}=1$
Using $\mathrm{y}(1)=3$,
$\mathrm{C}_{2}=3 \mathrm{e}-1$
Hence, from equation (i),
$\mathrm{y}=[1+(3 \mathrm{e}-1) x] \mathrm{e}^{-x}$
$y=e^{-x}+(3 e-1) x e^{-x}$
27. The value of $\int_{0}^{1} e^{x} d x$ using the trapezoidal rule with four equal subintervals is
(a) 1.718
(b) 1.727
(c) 2.192
(d) 2.718

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Ans. b

Exp:
Given, $\mathrm{n}=4, \mathrm{a}=0, \mathrm{~b}=1$
Then, $\quad h=\frac{b-a}{n}=0.25$
$\therefore \quad \mathrm{y}=\mathrm{f}(\mathrm{n})=\mathrm{e}^{\mathrm{x}}$

So,

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| :---: | :---: |
| 0 | $\mathrm{y}_{0}=1$ |
| $\frac{1}{4}$ | $\mathrm{y}_{1}=\mathrm{e}^{1 / 4}$ |
| $\frac{1}{2}$ | $\mathrm{y}_{2}=\mathrm{e}^{1 / 2}$ |
| $\frac{3}{4}$ | $\mathrm{y}_{3}=\mathrm{e}^{3 / 4}$ |
| 1 | $\mathrm{y}_{4}=\mathrm{e}^{1}$ |

Using T-rule,
$I=\int_{0}^{1} e^{n} d x=\frac{h}{2}\left[y_{o}+y_{4}+2\left(y_{1}+y_{2}+y_{3}\right)\right]$
$=\frac{0.25}{2}\left[1+e+2\left(e^{1 / 4}+e^{1 / 2}+e^{3 / 4}\right)\right]=1.727$
28. A 50 ml sample of industrial wastewater is taken into a silica crucible. The empty weight of the crucible is 54.352 g . The crucible with the sample is dried in a hot air oven at $104^{\circ} \mathrm{C}$ till a constant weight of 55.129 g . Thereafter, the crucible with the dried sample is fired at $600^{\circ} \mathrm{C}$ for 1 h in a muffle furnace, and the weight of the crucible along with residue is determined as 54.783 g . The concentration of total volatile solids is $\qquad$ _.
(a) $15540 \mathrm{mg} / \mathrm{l}$
(b) $8620 \mathrm{mg} / \mathrm{l}$
(c) $6920 \mathrm{mg} / \mathrm{l}$
(d) $1700 \mathrm{mg} / \mathrm{l}$

Ans. c

Exp:

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Volatile solids weight $=55.129-57.783=0.346 \mathrm{gm}($ in 50 ml$)$
So, $\quad$ in one litre $=\frac{346}{0.05}=6920 \mathrm{mg} /$ litre
29. A wedge M and a block N are subjected to force P and Q as shown in the figure. If force P is sufficiently large, then the block N can be raised. The weights of the wedge and the block are negligible compared to the forces $P$ and $Q$. The coefficient of friction $(\mu)$ along the inclined surface between the wedge and the block is 0.2 . All other surfaces are frictionless. The wedge angle is $30^{\circ}$.


The limiting force P , in terms of Q , required for impending motion of block N to just move it in the upward direction is given as $\mathrm{P}=\alpha \mathrm{Q}$. The value of the coefficient ' $\alpha$ ' (round off to one decimal place) is
(a) 0.6
(b) 0.5
(c) 2.0
(d) 0.9

Ans. d

Exp:

$\sum \mathrm{Y}=0$
$\mathrm{N}_{2} \sin 60^{\circ}-0.2 \mathrm{~N}_{2} \sin 30^{\circ}-\mathrm{Q}=0$
$\mathrm{Q}=0.766 \mathrm{~N}_{2}$
F.B.D. of wedge
$30^{\circ}$
$\Sigma \mathrm{X}=0$
$\Rightarrow \quad 0.2 \mathrm{~N}_{2} \cos 30^{\circ}+\mathrm{N}_{2} \cos 60^{\circ}-\mathrm{P}=0$
$\mathrm{P}=0.67 \mathrm{~N}_{2}$
$\Rightarrow \quad P=0.67 \times \frac{Q}{0.766}$
$\mathrm{P}=0.875 \mathrm{Q} \approx 0.9 \mathrm{Q}$
$\mathrm{P}=\alpha \mathrm{Q}$
$\Rightarrow \quad \alpha=0.9$
30. Contractor X is developing his bidding strategy against Contractor Y . The ratio of Y 's bid price to X 's cost for the 30 previous bids in which Contractor X has completed against Contractor Y is given in the table.

| Ratio of Y's bid price to $X$ 's cost | Number of bids |
| :---: | :---: |
| 1.02 | 6 |
| 1.04 | 12 |
| 1.06 | 3 |
| 1.10 | 6 |
| 1.12 | 3 |

Based on the bidding behaviour of the Contractor Y , the probability of winning against Contractor Y at a mark up of $8 \%$ for the next project is
(a) $0 \%$
(b) more than $0 \%$ but less than $50 \%$
(c) more than $50 \%$ but less than $100 \%$
(d) $100 \%$

Ans. b
31. Based on drained triaxial shear tests on sands and clays, the representative variations of volumetric strain $(\Delta \mathrm{V} / \mathrm{V})$ with the shear $\operatorname{strain}(\gamma)$ is shown in the figure.


Choose the correct option regarding the representative behaviour exhibited by Curve P and curve Q .
(a) Curve P represents dense sand and overconsolidated clay, while Curve Q represents loose sand and normally consolidated clay.
(b) Curve P represents dense sand and normally consolidated clay, while Curve Q represents loose sand and overconsolidated clay.

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(c) Curve P represents loose sand and overconsolidated clay, while Curve Q represents dense sand and normally consolidated clay.
(d) Curve P represents looses sand and normally consolidated clay, while Curve Q represents dense sand and over consolidated clay.

Ans. a

32. A fluid flowing steadily in a circular pipe of radius $R$ has a velocity that is everywhere parallel to the axis (centreline) on the pipe. The velocity distribution along the radial direction is $V_{r}=U\left(1-\frac{r^{2}}{R^{2}}\right)$, where r is the radial distance as measured from the pipe axis and U is the maximum velocity at $\mathrm{r}=0$. The average velocity of the fluid in the pipe is
(a) $\frac{U}{2}$
(b) $\frac{U}{3}$
(c) $\frac{U}{4}$
(d) $\left(\frac{5}{6}\right) U$

Ans. a
Exp:

$u=U\left(1-\frac{r^{2}}{R^{2}}\right)$
$\dot{m}=\int_{0}^{R} \rho(2 \pi r d r) u=2 \pi \rho U \int_{0}^{R}\left(1-\frac{r^{2}}{R^{2}}\right) r d r$
$\rho\left(\pi R^{2}\right) \bar{u}=2 \pi \rho U\left[\frac{R^{2}}{2}-\frac{R^{4}}{R^{2} \times 4}\right]$
$\bar{u}=\frac{U}{2}$
$\bar{u}=$ Mean velocity
$\mathrm{U}=\mathrm{Max}$ velocity
33. A water sample is analysed for coliform organisms by the multiple-tube fermentation method. The results of confirmed test are as follows:

| Sample size (ml) | Number of positive result out <br> of $\mathbf{5}$ tubes | Number of negative results <br> out of 5 tubes |
| :---: | :--- | :---: |
| 0.01 | 5 | 0 |
| 0.001 | 3 | 2 |
| 0.001 | 1 | 4 |

The most probable number (MPN) of coliform organisms for the above results is to be obtained using the following MPN index.

MPN Index for various Combination of Positive Results when Five Tubes used per dilution of 10.0 ml and 0.1 ml

| Combination of Positive Tubes | MPN index per 100 ml |
| :---: | :---: |
| $0-2-4$ | 11 |
| $1-3-5$ | 19 |
| $4-2-0$ | 22 |
| $5-3-1$ | 110 |

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(a) 1100000
(b) 110000
(c) 1100
(d) 110

Ans. b
Exp:
The sample size is $0.01 \mathrm{ml}, 0.001 \mathrm{ml}$ and 0.0001 ml which is 1000 times lesser than the standard of 10 $\mathrm{ml}, 1 \mathrm{ml}$ and 0.1 ml .

The positive set is 5-3-1 and w.r.t. the positive combination.
MPN $/ 100 \mathrm{ml}$ will be $110 \times 1000=110000$
34. Ammonia nitrogen is present in a given wastewater sample as the ammonium ion $\left(\mathrm{NH}_{4}{ }^{+}\right)$and ammonia $\left(\mathrm{NH}_{3}\right)$. If pH is the only deciding factor for the proportion of these two constituents, which of the following is a correct statement?
(a) At pH above 9.25 , only $\mathrm{NH}_{4}{ }^{+}$will be present.
(b) At pH below $9.25, \mathrm{NH}_{3}$ will be predominant
(c) At pH 7.0, $\mathrm{NH}_{4}{ }^{+}$and $\mathrm{NH}_{3}$ will be found in equal measures.
(d) At $\mathrm{pH} 7.0, \mathrm{NH}_{4}{ }^{+}$will be predominant

Ans. d
Exp:
From the curve given below, it is evident that at $\mathrm{pH} 7.0, \mathrm{NH}_{4}{ }^{+}$will be predominant.


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35. On a road, the speed-density relationship of a traffic stream is given by $u=70-0.7 \mathrm{k}$ (where, speed, u , is in $\mathrm{km} / \mathrm{h}$ and density, k , is in veh/km). At the capacity condition, the average time headway will be
(a) 0.5 s
(b) 1.0 s
(c) 1.6 s
(d) 2.1 s

Ans. d
Exp:
$\mathrm{u}=70-0.07 \mathrm{k}$
$u=70\left[1-\frac{k}{\frac{70}{0.7}}\right]$
$\mathrm{V}_{\mathrm{f}}=70 \mathrm{kmph}$
$k_{j}=\frac{70}{0.7}=100 \mathrm{veh} / \mathrm{km}$
$q_{\text {max }}=\frac{1}{4} V_{t} k_{j}=\left(\frac{1}{4} \times 70 \times 100\right)=1750 \mathrm{veh} / \mathrm{hr}$
$q_{\text {max }}=\frac{3600}{h_{i}}$
$1750=\frac{300}{h_{i}}$
Average time headway,
$h_{i}=\frac{3600}{1750} \approx 2.1 \mathrm{sec}$
Q. 36 - Q. 55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).
36. The values of abscissa ( $x$ ) and ordinate (y) of a curve are as follows:

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| $x$ | $y$ |
| :--- | :--- |
| 2.0 | 5.00 |
| 2.5 | 7.25 |
| 3.0 | 10.00 |
| 3.5 | 13.25 |
| 4.0 | 17.00 |

By Simpson's $1 / 3^{\text {rd }}$ rule, the area under the curve (round off to two decimal places) is $\qquad$ _.

Ans. $20.00-21.00$
Exp:
$\mathrm{d}=0.5$ unit
$\longrightarrow y_{1}=5$
$\rightarrow y_{2}=7.25$
$\rightarrow y_{3}=10$
$\rightarrow y_{4}=13.25$
$\rightarrow y_{5}=17$
$A=\frac{d}{3}\left[\left(y_{1}+y_{5}\right)+4\left(y_{2}+y_{4}\right)+2 y_{3}\right]$
$=\frac{0.5}{3}[(5+17)+4(7.25+13.25)+2 \times 10]$
$=20.67$ unit $^{2}$
37. Vehicular arrival at an isolated intersection follows the Poisson distribution. The mean vehicular arrival rate is 2 vehicle per minute. The probability (round off to two decimal places) that at least 2 vehicles will arrive in any given 1-minute interval is $\qquad$ _.

Ans. $0.58-0.60$

## Exp:

$\lambda=2 \mathrm{veh} / \mathrm{min}=2 \mathrm{veh} / 60 \mathrm{sec}=\frac{1}{30} \mathrm{vps}$
$\mathrm{n}=2, \mathrm{t}=1 \mathrm{~min}$
$P(n, t)=\frac{(\lambda t)^{n} e^{-\lambda t}}{n!}$

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$=\frac{(2 \times 1)^{2} \times e^{-(2 \times 1)}}{2!}=0.2706$
38. Refer the truss as shown in the figure (not to scale).


If load, $\mathrm{F}=10 \sqrt{3} \mathrm{kN}$, moment of inertia, $\mathrm{I}=8.33 \times 10^{6} \mathrm{~mm}^{4}$, area of cross section, $\mathrm{A}=10^{4} \mathrm{~mm}^{2}$ and length, $\mathrm{L}=2 \mathrm{~m}$ for all the members of the truss, the compressive stress (in $\mathrm{kN} / \mathrm{m}^{2}$, in integer) carried by the member $\mathrm{Q}-\mathrm{R}$ is $\qquad$ -

Ans. $490-510$
Exp:

$\mathrm{V}_{\mathrm{P}}=\mathrm{V}_{\mathrm{S}}=5 \sqrt{3} \mathrm{kN}$
Considering equilibrium of LHS of section (1)-(1):


Taking moment about 'T'
$\sum \mathrm{M}_{\mathrm{T}}(\mathrm{U})=0$
$(5 \sqrt{3} \times a)+F_{Q R}\left(\frac{\sqrt{3} a}{2}\right)=0$
$\Rightarrow \quad \mathrm{F}_{\mathrm{QR}}=-10 \mathrm{kN}$ or $10 \mathrm{kN}(\mathrm{C})$
Compressive stress in member $\mathrm{QR}\left(\sigma_{\mathrm{C}}\right)$
$\sigma_{C}=\frac{F_{Q R}}{2 A}=\frac{10 \mathrm{kN}}{2 \times\left(10^{4} \times 10^{-6}\right) \mathrm{m}^{2}}=500 \mathrm{kN} / \mathrm{m}^{2}$
39. A prismatic cantilever prestressed concrete beam of span length, $\mathrm{L}=1.5 \mathrm{~m}$ has one straight tendon placed in the cross-section as shown in the following figure (not to scale). The total prestressed force of 50 kN in the tendon is applied at $\mathrm{d}_{\mathrm{o}}=50 \mathrm{~mm}$ from the top in the cross-section of width, $\mathrm{b}=200$ mm and depth, $\mathrm{d}=300 \mathrm{~mm}$.


If the concentrated load, $\mathrm{P}=5 \mathrm{kN}$, the resultant stress (in MPa, in integer) experienced at point ' Q ' will be $\qquad$ .

Ans. 0

## Exp:


$e=\frac{D}{2}-50=\frac{300}{2}-50=100 \mathrm{~mm}$
$\mathrm{DL}=0.2 \times 0.3 \times 1.0 \times 25=1.50 \mathrm{kN} / \mathrm{m}$
$\mathrm{P}=50 \mathrm{kN}=50,000 \mathrm{~N}$
$\mathrm{W}=5 \mathrm{kN}$
Maximum $\mathrm{BM}=\frac{w L^{2}}{2}+W L$
$=\frac{1.5 \times 1.5^{2}}{2}+5 \times 1.50=9.1875 \mathrm{kNm}$

$\frac{P}{A}=\frac{50,000}{200 \times 300}=0.833 \mathrm{~N} / \mathrm{mm}^{2}$
$\frac{P e}{Z}=\frac{50,000 \times 100}{200 \times \frac{300^{2}}{6}}=1.67 \mathrm{~N} / \mathrm{mm}^{2}$
$\frac{M}{Z}=\frac{9.1875 \times 10^{6}}{200 \times \frac{300^{2}}{6}}=3.0625 \mathrm{~N} / \mathrm{mm}^{2}$
Stress at Q,
$=\frac{P}{A}+\frac{P e}{Z}-\frac{M}{Z}=0.833+1.67-3.0625$
$=-0.56 \mathrm{~N} / \mathrm{mm}^{2}$ (Tensile)
40. A column is subjected to total load $(\mathrm{P})$ of 60 kN supported through a bracket connection, as shown in the figure (not to scale).


The resultant force in bolt R (in kN , round off to one decimal place) is $\qquad$ .

Ans. 27.0 - 29.0

Exp:
$F_{1}=\frac{P}{n}=\frac{60}{6}=10 \mathrm{kN}$
$F_{2}=\frac{P \cdot e}{\Sigma r_{i}^{2}} \times r_{R}=\frac{60 \times 100 \mathrm{~mm}}{4 \times 50^{2}+2 \times 40^{2}} \times 40$
$=\frac{200}{11} \mathrm{kN}$
$\mathrm{F}_{\mathrm{R}}=\mathrm{F}_{1}+\mathrm{F}_{2}=10+\frac{200}{11}=28.2 \mathrm{kN}$
41. Employ stiffness matrix approach for the simply supported beam s shown in the figure given below to calculate unknown displacement/rotations. Take length, $\mathrm{L}=8 \mathrm{~m}$; modulus of elasticity, $\mathrm{E}=$ $3 \times 10^{4} \mathrm{~N} / \mathrm{mm}^{2}$; moment of inertia, $\mathrm{I}=225 \times 10^{6} \mathrm{~mm}^{4}$.


The mid-span deflection of the beam (in mm , round off to integer) under $\mathrm{P}=100 \mathrm{kN}$ in downward direction will be $\qquad$ .

Ans. 100-130

Exp:


By stiffness matrix method,
Step-1: Generation of stiffness matrix

## Column-1


$k_{11}=\frac{3 E(2 I)}{(L / 2)^{3}}+\frac{3 E(I)}{(L / 2)^{3}}=\frac{72 E I}{L^{3}}$
$k_{21}=-\frac{3 E(2 I)}{(L / 2)^{2}}+\frac{3 E(I)}{(L / 2)^{2}}=-\frac{12 E I}{L^{2}}$

## Column-2

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$k_{22}=\frac{3 E(2 I)}{(L / 2)}+\frac{3 E(I)}{(L / 2)}=18 \frac{E I}{L}$
Stiffness matrix $[\mathrm{k}]=\left[\begin{array}{cc}72 \frac{E I}{L^{3}} & -\frac{12 E I}{L^{2}} \\ -\frac{12 E I}{L^{2}} & \frac{18 E I}{L}\end{array}\right]$
Step-2: Calculation of unknown Nodal displacement $\left(\Delta_{B}, \theta_{B}\right)$
Using $\quad[\mathrm{P}]=[\mathrm{k}][\Delta]$
$\Rightarrow \quad\left[\begin{array}{l}P \\ O\end{array}\right]=\left[\begin{array}{cc}72 \frac{E I}{L^{3}} & -\frac{12 E I}{L^{2}} \\ -\frac{12 E I}{L^{2}} & \frac{18 E I}{L}\end{array}\right]\left[\begin{array}{l}\Delta_{B} \\ \theta_{B}\end{array}\right]$
On solving,
$\Delta_{B}=\frac{P L^{3}}{64 E I}(\downarrow)$
$\theta_{B}=\frac{P L^{2}}{96 E I}(C W)$
$\Delta_{B}=\frac{\left(100 \times 10^{3}\right) \times(8000)^{3}}{64 \times 3 \times 10^{4} \times 225 \times 10^{6}}$
$=118.519 \mathrm{~mm} \approx 119 \mathrm{~mm}$
42. A square plate O-P-Q-R of a linear elastic material with sides 1.0 m is loaded in a state of plane stress. Under a given stress condition, the plate deforms to a new configuration O-P'-Q'-R' as shown in the figure (not to scale). Under the given deformation, the edges of the plate remain straight.


The horizontal displacement of the point $(0.5 \mathrm{~m}, 0.5 \mathrm{~m})$ in the plate $\mathrm{O}-\mathrm{P}-\mathrm{Q}-\mathrm{R}$ (in mm , round off to one decimal place) is $\qquad$ -.

Ans. $2.4-2.6$
Exp:

FDB of edge PQ


FBD of Mid line


So, horizontal displacement of the point ( $0.5 \mathrm{~m}, 0.5 \mathrm{~m}$ )
$=-2.5 \mathrm{~mm}+5 \mathrm{~mm}=2.5 \mathrm{~mm}$
43. A small project has 12 activities $-\mathrm{N}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}, \mathrm{T}, \mathrm{U}, \mathrm{V}, \mathrm{W}, \mathrm{X}, \mathrm{Y}$ and Z . The relationship among these activities and the duration of these activities are given in the table.

| Activity | Duration (in weeks) | Depends upon |
| :---: | :---: | :---: |
| N | 2 | - |
| P | 5 | N |
| Q | 3 | N |
| R | 4 | P |
| S | 5 | Q |
| T | 8 | R |
| U | 7 | $\mathrm{R}, \mathrm{S}$ |
| V | 2 | U |
| W | 3 | U |
| X | 5 | $\mathrm{~T}, \mathrm{~V}$ |


| Y | 1 | W |
| :---: | :---: | :---: |
| Z | 3 | $\mathrm{X}, \mathrm{Y}$ |

The total float of the activity "V" (in weeks, in integer) is $\qquad$ .

Ans. 0
Exp:

$\mathrm{FTV}=\mathrm{T}_{\mathrm{Lj}}-\mathrm{T}_{\mathrm{Ei}}-\mathrm{t}_{\mathrm{ij}}$
$=20-18-2=0$
44. The soil profile at a construction site is shown in the figure (not to scale). Ground water table (GWT) is at 5 m below the ground level at present. An old well data shows that the ground water table was as low as 10 m below the ground level in the past. Take unit weight of water $\gamma_{\mathrm{w}}=9.81 \mathrm{kN} / \mathrm{m}^{3}$.


The over consolidation ratio (OCR) (round off to two decimal places) at the mid-point of the clay layer is $\qquad$ _.

Ans. 1.18-1.26

Exp:

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$O C R=\frac{\bar{\sigma}_{c}}{\bar{\sigma}_{o}}$
$\bar{\sigma}_{c}=$ Preconsolidation stress
$\bar{\sigma}_{o}=$ Present effective stress
$\bar{\sigma}_{c}=17.5 \times 10+(18.5-9.81) \times 5+(17-9.81) \times 4=247.21 \mathrm{kN} / \mathrm{m}^{2}$
$\bar{\sigma}_{o}=17.5 \times 5+(18.5-9.81) \times 10+(17-9.81) \times 4=203.16 \mathrm{kN} / \mathrm{m}^{2}$
$O C R=\frac{247.21}{203.16}=1.22$
45. A retaining wall of height 10 m with clay backfill is shown in the figure (not to scale). Weight of the retaining wall is 5000 kN per m acting at 3.3 from the toe of the retaining wall. The interface friction angle between base of the retaining wall and the base soil is $20^{\circ}$. The depth of clay in front of the retaining wall is 2.0 m . The properties of the clay backfill and the clay placed in front of the retaining wall are the same. Assume that the tension crack is filled with water. Use Rankine's earth pressure theory. Take unit weight of water, $\gamma_{w}=9.81 \mathrm{kN} / \mathrm{m}^{2}$.


## 5000 kN

The factor of safety (round off to two decimal places) against sliding failure of the retaining wall after ignoring the passive earth pressure will be $\qquad$ .

Ans. $4.20-4.35$
Exp:

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$Z_{o}=\frac{2 c}{\gamma \sqrt{k_{a}}}$
From $\phi=0, k_{a}=1$
$\Rightarrow \quad Z_{o}=\frac{2 \times 30}{17.2 \times \sqrt{1}}=3.49 \mathrm{~m}$


At $Z=10 \mathrm{~m}$
$\mathrm{P}_{\mathrm{a}}=\mathrm{k}_{\mathrm{a}} \gamma_{\text {sub }} 10-2 \mathrm{c} \sqrt{k_{a}}$
$=1 \times(17.2-9.81) \times 10-2 \times 30 \times \sqrt{1}=13.9$
$F_{a}=\frac{1}{2} \times 13.9 \times(10-3.49)=45.24 \mathrm{kN} / \mathrm{m}$
$F_{w}=\frac{1}{2} \times \gamma_{w} \times 10 \times 10=490.5 \mathrm{kN} / \mathrm{m}$
$\mathrm{N}=\mathrm{w}=5000 \mathrm{kN}$
$\mathrm{F}_{\mathrm{S}}=\mu \mathrm{N}=\tan \delta \times \mathrm{N}$

$$
\therefore \quad F O S=\frac{F_{s}}{F_{a}+F_{w}}=\frac{1819.85}{490.5+45.24}=3.39
$$

46. A combined trapezoidal footing of length $L$ supports two identical square columns ( $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ ) of size $0.5 \mathrm{~m} \times 0.5 \mathrm{~m}$, as shown in the figure. The columns $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ carry loads of 2000 kN and 1500 kN , respectively.


If the stress beneath the footing is uniform, the length of the combined footing L (in m , round off to two decimal places) is $\qquad$ -.

Ans. 5.70-5.90

Exp:
C.G. of load from $\mathrm{P}_{1}$

$P_{R} \bar{x}=P_{1} \times 0 \times P_{2} \times 5$
$\bar{x}=\frac{1500 \times 5}{3500}=2.143 \mathrm{~m}$
Distance of C.G. of footing from face of $\mathrm{P}_{1}$
$\bar{y}=\bar{x}+0.25=2.393 \mathrm{~m}$

C.G. of footing, $\bar{y}=\left(\frac{B_{1}+2 B_{2}}{B_{1}+B_{2}}\right) \times \frac{L}{3}$

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$2.393=\left(\frac{5+2 \times 1.5}{5+1.5}\right) \times \frac{L}{3}$
$\mathrm{L}=5.833 \mathrm{~m}$ say 5.83 m
47. An unsupported slope of height 15 m is shown in the figure (not to scale), in which the slope face makes an angle $50^{\circ}$ with the horizontal. The slope material comprises purely cohesive soil having undrained cohesion 75 kPa . A trial slip circle KLM with a radius 25 m , passes through the crest and toe of the slope and it subtends an angle $60^{\circ}$ at its centre O . The weight of the active soil mass ( W , bounded by KLMN) is $2500 \mathrm{kN} / \mathrm{m}$, which is acting at a horizontal distance of 10 m from toe of the slope. Consider the water table to be present at a very large depth from the ground surface.


Considering the trial slip circle KLM, the factor of safety against the failure of slope under undrained condition (round off to two decimal places) is $\qquad$ _.

Ans. 1.94-1.98
Exp:

$F O S=\frac{\text { Resisting moment }}{\text { Actuating moment }}$

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$F O S=\frac{C_{u} I R}{w \bar{x}}$
$l=$ Length of ac KLM
$\bar{x}=$ Distance of ' w ' from toe
Now, $\quad F O S=\frac{75 \times 2 \pi \times 25 \times \frac{60}{360} \times 25}{2500 \times 10}=1.96$
48. An unlined canal under regime conditions along with a silt factor of 1 has a width of flow 71.25 m . Assuming the unlined canal as a wide channel, the corresponding average depth of flow (in m , round off to two decimal places) in the canal will be $\qquad$ .

Ans: $2.80-2.95$
Exp:

$\mathrm{R}=\mathrm{D}$ (for wide rectangular channel)
$A f^{2}=140\left(\frac{2}{5} f R\right)^{5 / 2}$
$(B D) f^{2}=140\left(\frac{2}{5} f \times D\right)^{5 / 2}$
$(71.25 \times D) \times 1=140\left(\frac{2}{5} \times 1 \times D\right)^{5 / 2}$
$D \times 0.5089=\left(\frac{2}{5}\right)^{5 / 2} \times(D)^{5 / 2}$
$\mathrm{D}=2.94 \mathrm{~m}$
49. A cylinder ( 2.0 m diameter, 3.0 m long and 25 kN weight) is acted upon by water on one side and oil (specific gravity $=0.8$ ) on other side as shown in the figure.


The absolute ratio of the net magnitude of vertical forces to the net magnitude of horizontal forces (round off to two decimal places) is $\qquad$ .

Ans. $0.35-0.40$

## Exp:



Net horizontal force $\left(\mathrm{F}_{\mathrm{H}}\right)$ due to liquids
$\mathrm{F}_{\mathrm{H}}=\mathrm{F}_{\mathrm{H} 1}-\mathrm{F}_{\mathrm{H} 2}$

$$
=\rho_{w} g \bar{h}_{1} A_{v 1}-\rho_{o i l} g \bar{h}_{2} A_{v 2}
$$

$=\left(10^{3}\right)(9.81)\left(1+\frac{2}{2}\right)(2 \times 3)-(800)(9.81)\left(\frac{1}{2}\right)(1 \times 3)$
$\mathrm{F}_{\mathrm{H}}=105.948 \mathrm{kN}(\rightarrow)$
Net vertical force $\left(\mathrm{F}_{\mathrm{v}}\right)$ due to liquids
$\mathrm{F}_{\mathrm{V}}=\mathrm{F}_{\mathrm{V} 1}+\mathrm{F}_{\mathrm{V} 2}$
$=\rho_{\mathrm{w}} \mathrm{g} \forall_{1}+\rho_{\text {oil }} g \forall_{2}$
$=\left(10^{3}\right)(9.81)\left(\frac{\pi(1)^{2} \times 3}{2}\right)+(800)(9.81)\left(\frac{\pi}{4}(1)^{2} \times 3\right)$
$=64.7199 \mathrm{kN}(\uparrow)$

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$\frac{F_{V}}{F_{H}}=\frac{64.7199}{105.948}=0.61$
50. A tube-well of 20 cm diameter fully penetrates a horizontal, homogeneous and isotropic confined aquifer of infinite horizontal extent. The aquifer is of 30 m uniform thickness. A steady pumping at the rate of 40 litres/s from the well for a long time results in a steady drawdown of 4 m at the well face. The subsurface flow to the well due to pumping is steady, horizontal and Darcian and the radius of influence of the well is 245 m . The hydraulic conductivity of the aquifer (in m/day, round off to integer) is $\qquad$ .

Ans. 34-38
Exp:

$40 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{s}=\frac{2 \pi k \times 30 \mathrm{~m} \times 4 \mathrm{~m}}{\ln \left(\frac{245}{0.1}\right)}$
$\mathrm{k}=4.14 \times 10^{-4} \mathrm{~m} / \mathrm{s}$
$\mathrm{k}=35.77 \mathrm{~m} / \mathrm{d} \approx 36 \mathrm{~m} / \mathrm{d}$
51. A baghouse filter has to treat $12 \mathrm{~m}^{3} / \mathrm{s}$ of waste gas continuously. The baghouse is to be divided into 5 sections of equal cloth area such that one section can be shut down for cleaning and/or repairing, while the other 4 sections continue to operate. An air-to-cloth ratio of $6.0 \mathrm{~m}^{3} / \mathrm{min}-\mathrm{m}^{2}$ cloth will provide sufficient treatment to the gas. The individual bags are of 32 cm in diameter and 5 m in length. The total number of bags (in integer) required in the baghouse is $\qquad$ .

Ans. 30
Exp:
Given, discharge to be passed $=12 \mathrm{~m}^{3} / \mathrm{s}$
Out of 5 sections, 4 will operate at a time.

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Each bag is


Thus, surface area of each bag $=\pi \times 0.32 \times 5 \mathrm{~m}^{2}=5.0265 \mathrm{~m}^{2}$
Total surface area required w.r.t. given discharge
$=\frac{12 \mathrm{~m}^{3} / \mathrm{s}}{6 \mathrm{~m}^{3} / \mathrm{min}-\mathrm{m}^{2}}=\frac{12 \mathrm{~m}^{3} / \mathrm{s}}{\frac{6}{60} \mathrm{~m}^{3} / \mathrm{s}-\mathrm{m}^{2}}=120 \mathrm{~m}^{2}$
$\therefore \quad$ Working bag filters required $=\frac{120 \mathrm{~m}^{2}}{5.0265 \mathrm{~m}^{2}}=23.87$ or 24 bags filters
But, since it is asked total (which includes a standby set also), then,
Total number of bag filters $=24 \times \frac{5}{4}=30$
52. A secondary clarifier handles a total flow of $9600 \mathrm{~m}^{3} / \mathrm{d}$ from the aeration tank of a conventional activated-sludge treatment system. The concentration of solids in the flow from the aeration tank is $3000 \mathrm{mg} / \mathrm{l}$. The clarifier is required to thicken the solids to $12000 \mathrm{mg} / \mathrm{l}$, and hence it is to be designed for a solid flux of $3.2 \mathrm{~kg} / \mathrm{m}^{2} \mathrm{~h}$. The surface area of the designed clarifier for thickening (in $\mathrm{m}^{2}$, in integer) is $\qquad$ .

Ans. 375
Exp:
$\mathrm{Q}=9600 \mathrm{~m}^{3} / \mathrm{d}$
$\mathrm{X}=3000 \mathrm{mg} / \mathrm{l}$
$\mathrm{X}_{\mathrm{u}}=12000 \mathrm{mg} / \mathrm{l}$
Solids loading rate or solid flux $=3.2 \mathrm{~kg} / \mathrm{m}^{2}-\mathrm{h}$

Surface area, $A=\frac{\text { Total quantity of solids entering }}{\text { Solid flux }}$
$A=\frac{9600 \mathrm{~m}^{3} / \mathrm{d} \times 3000 \mathrm{mg} / \mathrm{l}}{3.2 \times 10^{6} \mathrm{mg} / \mathrm{m}^{2}-\mathrm{h}}$
$=\frac{9600 \times 10^{3} \mathrm{l} / \mathrm{d} \times 3000 \mathrm{mg} / \mathrm{l}}{3.2 \times 10^{6} \mathrm{mg} / \mathrm{m}^{2}-\mathrm{h} \times 24 \mathrm{~h} / \mathrm{d}}$
$=375 \mathrm{~m}^{2}$
53. Spot speeds of vehicles observed at a point on a highway are $40,55,60,65$ and $80 \mathrm{~km} / \mathrm{h}$. The space-mean speed (in $\mathrm{km} / \mathrm{h}$, round off to two decimal places) of the observed vehicles is $\qquad$
Ans. 55.50-58.50
Exp:
$\mathrm{V}_{1}=40, \mathrm{~V}_{2}=55, \mathrm{~V}_{3}=60, \mathrm{~V}_{4}=65, \mathrm{~V}_{5}=80$
$V_{S}=\left(\frac{n}{\frac{1}{V_{1}}+\frac{1}{V_{2}}+\ldots+\frac{1}{V_{n}}}\right)=\frac{50}{\frac{1}{40}+\frac{1}{55}+\frac{1}{60}+\frac{1}{65}+\frac{1}{80}}=56.99 \mathrm{kmph}$
54. The longitudinal section of a runway provides the following data:

| End-to-end runway (m) | Gradient (\%) |
| :---: | :---: |
| 0 to 300 | +1.2 |
| 300 to 600 | -0.7 |
| 600 to 1100 | +0.6 |
| 1100 to 1400 | -0.8 |
| 1400 to 1700 | -1.0 |

The effective gradient of the runway (in $\%$, round off to two decimal places) is $\qquad$ .

Ans. $0.30-0.34$

## Exp:

Assuming RL of start of runway as datum, i.e., $\mathrm{RL}=0 \mathrm{~m}$ )

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Effective gradient $=\left[\frac{\text { Maximum difference in reduced level }}{\text { Total runwaylength }}\right]$
$=\left[\frac{4.5-(-0.9)}{1700} \times 100\right] \%$
$=0.3176 \% \approx 0.32 \%$
55. Traversing is carried out for a closed traverse PQRS. The internal angles at vertices $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ and S are measured a $92^{\circ}, 68^{\circ}, 123^{\circ}$ and $77^{\circ}$, respectively. If fore bearing of line PQ is $27^{\circ}$, fore bearing of line Rs. (in degrees, in integer) is $\qquad$ _.

Ans. 196 - 196 or $218-218$
Exp:

$Q=\left[\begin{array}{l}B B \text { of } P Q=27^{\circ}+180^{\circ}=207^{\circ} \\ F B \text { of } Q R=207^{\circ}-68^{\circ}=139^{\circ}\end{array}\right.$
$R=\left[\begin{array}{l}B B \text { of } Q R=139^{\circ}+180^{\circ}=319^{\circ} \\ F B \text { of } R S=319^{\circ}-123^{\circ}=196^{\circ}\end{array}\right.$

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$$
\begin{aligned}
& Q=\left[\begin{array}{l}
B B \text { of } P Q=27^{\circ}+180^{\circ}=207^{\circ} \\
F B \text { of } Q R=207^{\circ}+68^{\circ}=275^{\circ}
\end{array}\right. \\
& R=\left[\begin{array}{l}
B B \text { of } Q R=275^{\circ}-180^{\circ}=95^{\circ} \\
F B \text { of } R S=95^{\circ}+123^{\circ}=218^{\circ}
\end{array}\right.
\end{aligned}
$$

