

GATE 2021

CHEMICAL ENGINEERING

APTITUDE

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: - 1/3).

1. The ratio of boys to girls in a class is 7 to 3.

Among the options below, an acceptable value for the total number of students in the class is :

- (a) 21
- (b) 37
- (c) 50
- (d) 73

Ans. c





Exp:

Sum of Ratio =  $7 + 3 = 10$

So, total number of students will be a multiple of 10.

Hence, only 1 option is available, i.e., option c.

2. A polygon is convex if, for every pair of points, P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon. Which one of the following is **NOT** a convex polygon?

- (a) 
- (b) 
- (c) 
- (d) 

Ans. a

Exp:

According to line segment in case of a line segment will fall outside the polygon. So, a should be the answer.

3. Consider the following sentences:

- (i) Everybody in the class is prepared for the exam.
- (ii) Babu invited Danish to his home because he enjoys playing chess.

Which of the following is the CORRECT observation about the above two sentences?

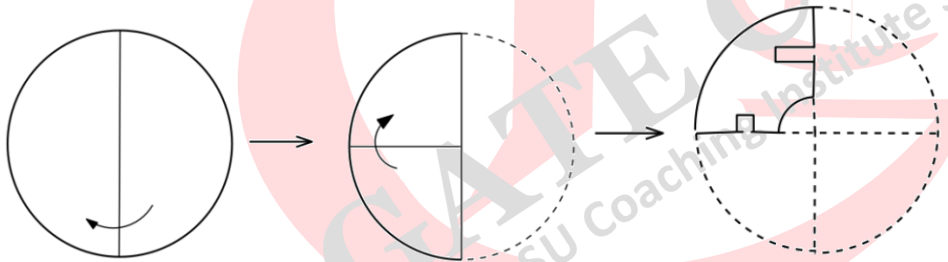
- (a) (i) is grammatically correct and (ii) is unambiguous
- (b) (i) is grammatically incorrect and (ii) is unambiguous
- (c) (i) is grammatically correct and (ii) is ambiguous
- (d) (i) is grammatically incorrect and (ii) is ambiguous

Ans. c

Exp:

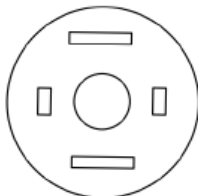
Everybody is singular and takes singular verb is, first statement is correct and the purpose of invitation is unclear so second statement is vague and that sounds ambiguous.

4.

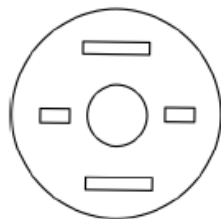


A circular sheet of paper is folded along the lines in the direction shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like

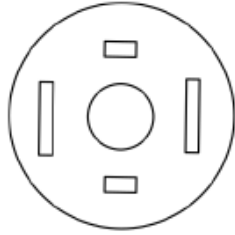
\_\_\_\_\_.



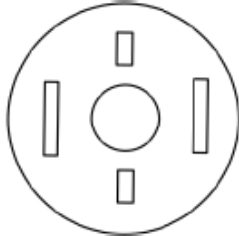
(a)



(b)



(c)



(d)

Ans. a

Exp:

As the paper is folded twice and as per question given if we will cut it into two places, then it will look like as figure no. a.

5. \_\_\_\_ is to surgery as writer is to \_\_\_\_

Which one of the following options maintains a similar logical relation in the above sentence?

(a) Plan, outline

(b) Hospital, library

(c) Doctor, book

(d) Medicine, grammar

Ans. c

Exp:

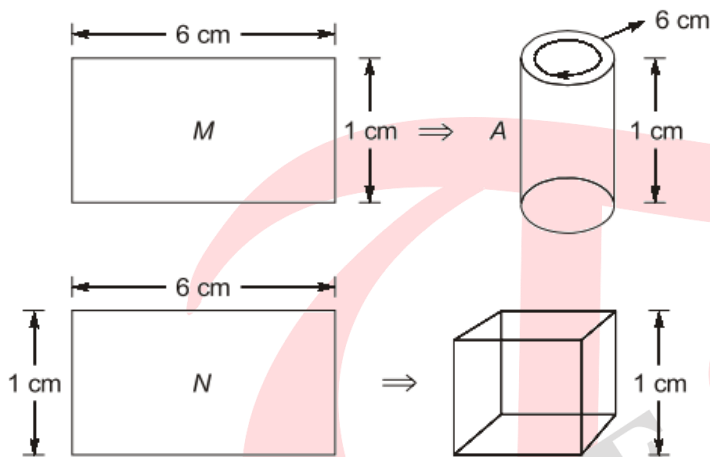
Doctor is known for surgery so as writer is known for writing books.

**Q. 6 - Q. 10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: - 2/3).**

6. We have 2 rectangular sheets of paper, M and N, of dimensions  $6 \text{ cm} \times 1 \text{ cm}$  each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is \_\_\_\_\_.

- (a)  $\frac{\pi}{2}$   
 (b)  $\frac{3}{\pi}$   
 (c)  $\frac{9}{\pi}$   
 (d)  $3\pi$   
 Ans. c

Exp:



Volume of cylinder =  $\pi r^2 h$

Now,  $2\pi r = 6$  (figure A)

$$r = \frac{3}{\pi}$$

$$\text{Volume of cylinder} = \pi \times \frac{3}{\pi} \times \frac{3}{\pi} \times 1 = \frac{9}{\pi}$$

Volume of cube =  $(1)^3$

$$\text{Ratio} = \frac{\frac{9}{\pi}}{1} = \frac{9}{\pi}$$

7.

Item	Cost Price	Profit %	Marked Price
P	5400	—	5860
Q	—	25	10000

Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3 : 4. Discount is calculated as the difference between the marked price and the

selling price. The profit percentage is calculated as the ratio of the difference between selling price and cost, to the cost  $\left( \text{Profit \%} = \frac{\text{selling price} - \text{Cost}}{\text{Cost}} \times 100 \right)$ .

The discount on item Q, as a percentage of its marked price, is \_\_\_\_\_.

- (a) 25
- (b) 12.5
- (c) 10
- (d) 5

Ans. c

Exp:

$$CP_p = 5400, \quad MP_p = 5860$$

$$\frac{5400 \times 4}{5} = CP_Q \quad \Rightarrow \quad CP_Q = 7200$$

$$SP_Q = ?$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100 \text{ [for Q]}$$

$$25 = \left( \frac{SP - 7200}{7200} \right) \times 100$$

$$SP_Q = 9000$$

$$\text{Discount Q} = MP_Q - SP_Q = 1000$$

$$\text{Discount \%} = \frac{1000}{10,000} \times 100 = 10\%$$

8. There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag. The probability that at least two chocolates are identical is \_\_\_\_\_

- (a) 0.3024
- (b) 0.4235
- (c) 0.6976
- (d) 0.8125

Ans. c

Exp:

$$\text{Total number of ways to take out the chocolates are} = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

Number of ways in which no chocolate will be identical =  $10 \times 9 \times 8 \times 7 \times 6 = 30240$

Probability in which no chocolate will be identical =  $30240/10^5$

Probability in which at least two chocolate are identical =  $1 - (30240/10^5) = 0.6976$

9. Given below are two statements 1 and 2, and two conclusions I and II.

Statement 1: All bacteria are microorganisms.

Statement 2: All pathogens are microorganisms.

Conclusion I: Some pathogens are bacteria.

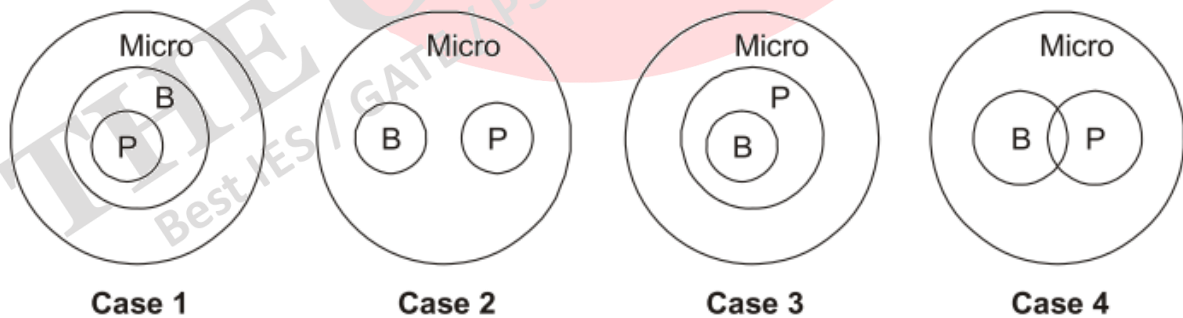
Conclusion II: All pathogens are not bacteria.

Based on the above statements and conclusions, which one of the following options is logically CORRECT?

- (a) Only conclusion I is correct
- (b) Only conclusion II is correct
- (c) Either conclusion I or II is correct.
- (d) Neither conclusion I nor II is correct.

Ans. c or d

Exp:



None of the two conclusions will satisfy all the 4 cases.

10. Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality.

Which one of the following statements summarizes the passage?

- (a) The proposed AOM addresses the core problems that cause obesity.

(b) If obesity reduces, poverty will naturally reduce, since obesity causes poverty.

(c) AOM are addressing the core problems and are likely to succeed.

(d) AOM are addressing the problem superficially.

Ans. d

Exp:

Superficially is the deciding key word which means apparently/seemingly.



## CHEMICAL ENGINEERING (CH)

**Q.1 – Q.15 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: - 1/3).**

1. An ordinary differential equation (ODE),  $\frac{dy}{dx} = 2y$ , with an initial condition  $y(0) = 1$ , has the analytical solution  $y = e^{2x}$ .

Using Runge-Kutta second order method, numerically integrate the ODE to calculate  $y$  at  $x = 0.5$  using a step size of  $h = 0.5$ .

If the relative percentage error is defined as,

$$\varepsilon = \left| \frac{y_{\text{analytical}} - y_{\text{numerical}}}{y_{\text{analytical}}} \right| \times 100$$

Then the value of  $\varepsilon$  at  $x = 0.5$  is \_\_\_\_\_.

- (a) 0.06
- (b) 0.8
- (c) 4.0
- (d) 8.0

Ans. d

Exp:

$$y_{n+1} = y_n + \frac{1}{2} [k_1 + k_2]$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + h, y_n + k_1)$$

$$x_0 = 0, x_0 + h = 0 + 0.5 = 0.5 = x_1$$

$$y_0 = 1, y_1 = y_0 + \frac{1}{2} [k_1 + k_2]$$

$$y_{0.5} = 1 + \frac{1}{2} [k_1 + k_2]$$

$$k_1 = 0.5 \times (2 \times 1) = 1 \text{ \& } k_2 = 0.5 \times (1 + 1) = 2$$

$$y_{0.5} = 1 + \frac{1}{2} [1 + 2] = 2.5 \quad \text{Numerical Solution}$$

Analysis,

$$y_{0.5} = e^{2 \times 0.5}$$

$$= e^1 = 2.73$$



$$\% \text{ error} = \left( \frac{2.73 - 2.5}{2.73} \right) \times 100 \approx 8.05\% = \epsilon$$

2. The function  $\cos(x)$  is approximated using Taylor series around  $x = 0$  as

$\cos(x) \approx 1 + a x + b x^2 + c x^3 + d x^4$ . The values of  $a, b, c$  and  $d$  are

(a)  $a = 1, b = -0.5, c = -1, d = -0.25$

(b)  $a = 0, b = -0.5, c = 0, d = 0.042$

(c)  $a = 0, b = 0.5, c = 0, d = 0.042$

(d)  $a = -0.5, b = 0, c = 0.042, d = 0$

Ans. b

Exp:

Expansion of  $\cos x$  from Taylor series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$$

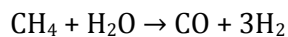
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

On comparing with  $\cos x$  form given above, we have

$$\cos(x) \approx 1 + a x + b x^2 + c x^3 + d x^4$$

$$a = 0, b = -\frac{1}{2!} = -0.5, c = 0, d = 0.04167 \approx 0.042$$

3. The heat of combustion of methane, carbon monoxide and hydrogen are  $P, Q$  and  $R$  respectively. For the reaction below,



The heat of reaction is given by

(a)  $P - Q - 3R$

(b)  $Q + 3R - P$

(c)  $P - Q - R$

(d)  $Q + R - P$

Ans. a

Exp:

As we know for water (H<sub>2</sub>O) molecule, standard heat of combustion is zero.

&

Standard heat of reaction is given by:

$$\begin{aligned}\Delta H_{rxn}^{\circ} &= -\sum v_i \Delta H_{ci}^{\circ} \\ &= -\left[ v_{CH_4} \cdot \Delta H_{C_{CH_4}}^{\circ} + v_{CO} \cdot \Delta H_{C_{CO}}^{\circ} + v_{H_2} \cdot \Delta H_{C_{H_2}}^{\circ} \right] \\ &= -[(-1) \times P + (1) \times Q + 3 \times R] \\ &= P - Q - 3R\end{aligned}$$

4. A batch settling experiment is performed in a long column using a dilute dispersion containing equal number of particles of type A and type B in water (density 1000 kg/m<sup>3</sup>) at room temperature.

Type A are spherical particles of diameter 30 μm and density 1100 kg m<sup>-3</sup>.

Type B are spherical particles of diameter 10 μm and density 1900 kg m<sup>-3</sup>.

Assuming that Stoke's law is valid throughout the duration of the experiment, the settled bed would

(a) consist of a homogeneous mixture of type A and type B particles

(b) consist of type B particles only

(c) be completely segregated with type B particles on top of type A particles

(d) be completely segregated with type A particles on top of type B particles

Ans. a

Exp:

As there is no idea about the flow regime what was given.

**Approach:**

Just check for settling ratio or the ratio of terminal settling velocities.

$$\text{if } \frac{v_{tA}}{v_{tB}} > 1 \quad \Rightarrow \quad \text{'B' will be above 'A'}$$

$$\text{if } \frac{v_{tA}}{v_{tB}} < 1 \quad \Rightarrow \quad \text{'A' will be above 'B'}$$

$$\text{if } \frac{v_{tA}}{v_{tB}} = 1 \quad \Rightarrow \quad \text{'A' and 'B' will settle together so, homogeneous mixture.}$$

**Solution:**

For Stoke's law Regime, We know that,

$$v_t = \frac{dp^2 g (\rho_p - \rho_f)}{18\mu_f}$$

$$\Rightarrow \frac{v_{tA}}{v_{tB}} = \frac{dp_A^2 (\rho_{pA} - \rho_f)}{dp_B^2 (\rho_{pB} - \rho_f)}$$

$$= \frac{(30\mu m)^2 \times (1100 - 1000) \frac{kg}{m^3}}{(10\mu m)^2 \times (1900 - 1000) \frac{kg}{m^3}}$$

$$= 1$$

$$\Rightarrow v_{tA} = v_{tB}$$

So, mixture will be homogeneous with type 'A' & type 'B' particles.

5. A three-dimensional velocity field is given by  $V = 5x^2y \mathbf{i} + Cy \mathbf{j} - 10xyz \mathbf{k}$ , where  $\mathbf{i}, \mathbf{j}, \mathbf{k}$  are the unit vectors in  $x, y, z$  directions, respectively, describing a Cartesian coordinate system. The coefficient  $C$  is a constant. If  $V$  describes an incompressible fluid flow, the value of  $C$  is

(a) -1

(b) 0

(c) 1

(d) 5

Ans. b

Exp:

For incompressible fluid, we know that divergence of velocity will be 0.

$$\text{i.e., } \nabla \cdot \vec{V} = 0$$

$$\Rightarrow \nabla \cdot \vec{V} = 0 \text{ means } \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

$$\text{As } \vec{V} = f \cdot x\hat{i} + g \cdot y\hat{j} + h \cdot z\hat{k}$$

$$\text{So, } \nabla \cdot \vec{V} = \frac{\partial}{\partial x}(5x^2y) + \frac{\partial}{\partial y}(cy) + \frac{\partial}{\partial z}(-10xyz)$$

$$0 = 10x \cdot y + c - 10xy$$

$$\Rightarrow c = 10x(y - y) = 0$$

6. Heat transfer coefficient for a vapor condensing as a film on a vertical surface is given by

(A) Dittus-Boelter equation

(B) Nusselt theory

(C) Chilton-Colburn analogy

(D) Sieder-Tate equation

Ans. b

7. In a double-pipe heat exchanger of 10 m length, a hot fluid flows in the annulus and a cold fluid flows in the inner pipe. The temperature profiles of the hot ( $T_h$ ) and cold ( $T_c$ ) fluids along the length of the heat exchanger ( $x$  such that  $x \geq 0$ ), are given by

$$T_h(x) = 80 - 3x$$

$$T_c(x) = 20 + 2x$$

where  $T_h$  and  $T_c$  are in  $^{\circ}\text{C}$  and  $x$  in meter.

The logarithmic mean temperature difference (in  $^{\circ}\text{C}$ ) is

(a) 24.6

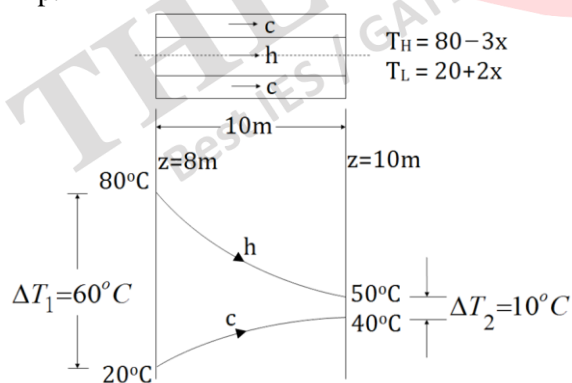
(b) 27.9

(c) 30.0

(d) 50.0

Ans. b

Exp:



$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

$$LMTD = \frac{60 - 10}{\ln \frac{60}{10}} = 27.9^{\circ}\text{C}$$

8. For a shell-and-tube heat exchanger, the clean overall heat transfer coefficient is calculated as  $250 \text{ W m}^{-2} \text{ K}^{-1}$  for a specific process condition. It is expected that the heat exchanger may be fouled during the operation, and a fouling resistance of  $0.001 \text{ m}^2 \text{ K W}^{-1}$  is prescribed. The dirt overall heat transfer coefficient is \_\_\_\_  $\text{W m}^{-2} \text{ K}^{-1}$ .

- (a) 100
- (b) 150
- (c) 200
- (d) 250

Ans. c

Exp:

$$\begin{aligned} \frac{1}{U_d} &= \frac{1}{U_c} + \text{resistance of fouling} \\ &= \frac{1}{250} + 0.001 \\ &= 0.005 \\ \therefore U_d &= 200 \text{ W/m}^2 \text{ K} \end{aligned}$$

#### Newly Added Topic

9. In reverse osmosis, the hydraulic pressure and osmotic pressure at the feed side of the membrane are  $P_1$  and  $\pi_1$ , respectively. The corresponding values are  $P_2$  and  $\pi_2$  at the permeate side. The membrane, feed, and permeate are at the same temperature. For equilibrium to prevail, the general criterion that should be satisfied is

- (a)  $\pi_1 = \pi_2$
- (b)  $P_1 = P_2$
- (c)  $P_1 + \pi_1 = P_2 + \pi_2$
- (d)  $P_1 - \pi_1 = P_2 - \pi_2$

Ans. d

Exp:

In Reverse osmosis, the NDP (Net Driving Pressure) refers to the difference between the feed pressure and the osmotic pressure. It is the measure of the actual driving pressure available to force the water through the membrane. The NDP available should be positive and constant.

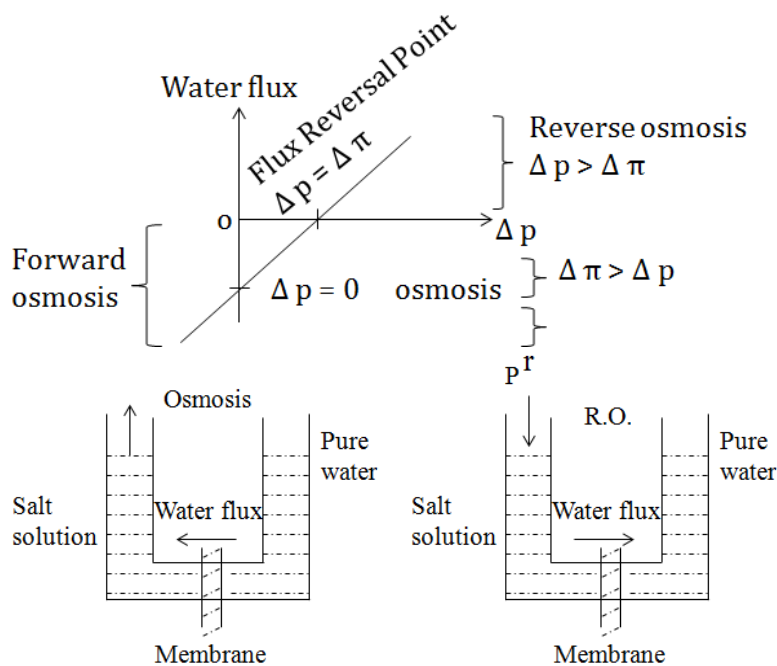
$$\text{NDP} = \Delta P_1 - \Delta P_2 \geq 0$$

$$\text{where, } \Delta P_1 = (P_1 - P_{i1})$$

$$\Delta P_2 = (P_2 - P_{i2})$$

$$\Rightarrow P_1 - \pi_1 = P_2 - \pi_2$$

So, option (d) is the correct answer.



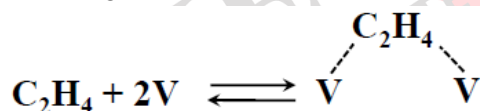
For R.O. to occur,

$$\Delta P \geq \Delta \pi$$

$$(P_1 - P_2) \geq (P_{i1} - P_{i2})$$

$$P_1 - P_{i1} \geq P_2 - P_{i2}$$

10. Ethylene adsorbs on the vacant active sites  $V$  of a transition metal catalyst according to the following mechanism.



If  $N_T$ ,  $N_V$  and  $N_{\text{C}_2\text{H}_4}$  denote the total number of active sites, number of vacant active sites and number of adsorbed  $\text{C}_2\text{H}_4$  molecules, respectively, the balance on the total number of active sites is given by

- (a)  $N_T = N_V + N_{\text{C}_2\text{H}_4}$
- (b)  $N_T = N_V + 2 N_{\text{C}_2\text{H}_4}$
- (c)  $N_T = 2N_V + N_{\text{C}_2\text{H}_4}$
- (d)  $N_T = N_V + 0.5N_{\text{C}_2\text{H}_4}$

Ans. b

Exp:

Total number of active sites = Total number of vacant site + Total number of occupied sites

$$\Rightarrow N_T = N_V + 2 N_{\text{C}_2\text{H}_4} \rightarrow \text{Number of occupied sites by } \text{C}_2\text{H}_4$$

$$N_T = N_V + 2 N_{\text{C}_2\text{H}_4}$$

11. Which of the following is NOT a standard to transmit measurement and control signals?

- (a) 4 – 20 mA
- (b) 3 – 15 psig
- (c) 0 – 100 %
- (d) 1 – 5 VDC

Ans. c

12. A feedforward controller can be used only if

- (a) the disturbance variable can be measured
- (b) the disturbance variable can be manipulated
- (c) the disturbance variable can be ignored
- (d) regulatory control is not required

Ans. a

13. Turnover ratio is defined as

- (a)  $\frac{\text{Fixed capital investment}}{\text{Gross annual sales}}$
- (b)  $\frac{\text{Gross annual sales}}{\text{Fixed capital investment}}$
- (c)  $\frac{\text{Fixed capital investment}}{\text{Average selling price of the product}}$
- (d)  $\frac{\text{Gross annual sales}}{\text{Average selling price of the product}}$

Ans. b

14. A principal amount is charged a nominal annual interest rate of 10%. If the interest rate is compounded continuously, the final amount at the end of one year would be

- (a) higher than the amount obtained when the interest rate is compounded monthly
- (b) lower than the amount obtained when the interest rate is compounded annually
- (c) equal to 1.365 times the principal amount

(d) equal to the amount obtained when using an effective interest rate of 27.18%

Ans. a

Exp:

$$(s = pe^m) \rightarrow \text{for Continuous compounding}$$

$$r = 0.1, n = 1$$

$$s \cong 1.105 P$$

$$i_{\text{eff}} = e^{r-1}$$

$$\cong 10.5 \%$$

15. Match the common name of chemicals in Group – I with their chemical formulae in Group – 2.

Group – I	Group – II
P. Gypsum	I. $\text{Ca}(\text{H}_2\text{PO}_4)_2$
Q. Dolomite	II. $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$
R. Triple Super Phosphate	III. $\text{CaCO}_3 \cdot \text{MgCO}_3$

The correct combination is:

(a) P – III, Q – II, R – I

(b) P – III, Q – I, R – II

(c) P – II, Q – III, R – I

(d) P – II, Q – I, R – III

Ans. c

**Q.16 – Q.18 Multiple Select Question (MSQ), carry ONE mark each (no negative marks).**

16. For the function  $f(x) = \begin{cases} -x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$

The CORRECT statement(s) is/are

(a)  $f(x)$  is continuous at  $x = 1$

(b)  $f(x)$  is differentiable at  $x = 1$

(c)  $f(x)$  is continuous at  $x = 0$

(d)  $f(x)$  is differentiable at  $x = 0$

Ans. a, b, c

Exp:

$$\lim_{x \rightarrow 0^-} = -ve$$

$$\lim_{x \rightarrow 0^+} = +ve$$



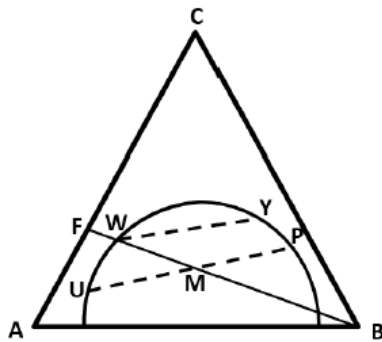
At  $x = 0$ , not differentiable.

At  $x = 1 \rightarrow$  continuous

$x = 0 \rightarrow$  continuous

$x = 1 \rightarrow$  differentiable

17. Feed solution F is contacted with solvent B in an extraction process. Carrier liquid in the feed is A and the solute is C. The ternary diagram depicting a single ideal stage extraction is given below. The dashed lines represent the tie-lines.

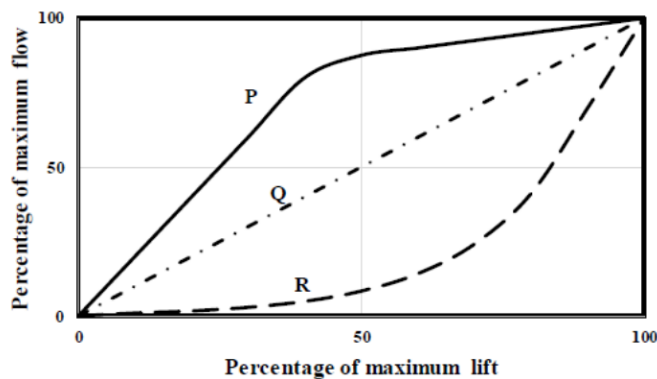


The correct option(s) is/are

- (a) For the tie-lines shown, concentration of solute in the extract is higher than that in the raffinate.
- (b) Maximum amount of solvent is required if the mixture composition is at W.
- (c) Y represents the composition of extract when minimum amount of solvent is used.
- (d) U represents the raffinate composition if the mixture composition is at M

Ans. a, c, d

18. The inherent characteristics of three control valves P, Q and R are shown in the figure.



The CORRECT option(s) is/are

- (a) P is a quick opening valve
- (b) Q is a quick opening valve
- (c) P is an equal percentage valve
- (d) R is an equal percentage valve

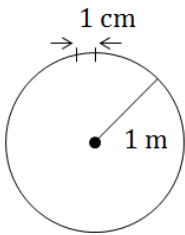
Ans. a, d

**Q.19 - Q.25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).**

Q.19 A source placed at the origin of a circular sample holder (radius  $r = 1 \text{ m}$ ) emits particles uniformly in all directions. A detector of length  $l = 1 \text{ cm}$  has been placed along the perimeter of the sample holder. During an experiment, the detector registers 14 particles. The total number of particles emitted during the experiment is \_\_\_\_\_.

Ans. 8790 – 8800

Exp:



$$\frac{2\pi \times 1\text{m}}{0.01} = 8796.46$$

20. A, B, C and D are vectors of length 4.

$$A = [a_1 \ a_2 \ a_3 \ a_4]$$

$$B = [b_1 \ b_2 \ b_3 \ b_4]$$

$$C = [c_1 \ c_2 \ c_3 \ c_4]$$

$$D = [d_1 \ d_2 \ d_3 \ d_4]$$

It is known that B is not a scalar multiple of A. Also, C is linearly independent of A and B. Further,  $D = 3A + 2B + C$ .

The rank of the matrix

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \text{ is } \underline{\hspace{2cm}}$$

Ans. 3

Exp:

$$f(A) = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix}$$

Now,

$$R_3 \rightarrow R_3 - (3R_1 + 2R_2 + R_3)$$

$$R_3 \rightarrow 0$$

$$f(A) = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows = 3.

$$f(A) = 3$$

21. The van der Waals equation of state is given by

$$P_r = \frac{8T_r}{3v_r - 1} - \frac{3}{v_r^2}$$

where  $P_r$ ,  $T_r$  and  $v_r$  represent reduced pressure, reduced temperature and reduced molar volume, respectively. The compressibility factor at critical point ( $z_c$ ) is  $3/8$ .

If  $v_r = 3$  and  $T_r = 4/3$ , then the compressibility factor based on the van der Waals equation of state is \_\_\_\_\_ (round off to 2 decimal places).

Ans. 0.83 – 0.85

Exp:

$$P_r = \frac{8T_r}{3v_r - 1} - \frac{3}{v_r^2}$$

$$z_c = \frac{3}{8}$$

$$v_r = 3 \text{ and } T_r = \frac{4}{3}, z = ?$$

$$P_r = \frac{8 \times \frac{4}{3}}{9 - 1} - \frac{3}{9} = 1$$

$$z_c = \frac{P_c v_c}{T_c} = \frac{P}{P_r} \cdot \frac{v}{v_r} \cdot \frac{T_r}{T} = \frac{Pv}{T} \cdot \frac{T_r}{P_r v_r}$$

$$\frac{3}{8} = z \cdot \frac{4/3}{1 \times 3} = \frac{4}{9} z$$

$$z = \frac{3}{8} \times \frac{9}{4} = 0.84375 \approx 0.84$$

22. Consider a steady flow of an incompressible, Newtonian fluid through a smooth circular pipe. Let  $\alpha_{\text{laminar}}$  and  $\alpha_{\text{turbulent}}$  denote the kinetic energy correction factors for laminar and turbulent flow through the pipe, respectively. For turbulent flow through the pipe

$$\alpha_{\text{turbulent}} = \left( \frac{V_o}{\bar{V}} \right)^3 \frac{2n^2}{(3+n)(3+2n)}$$

Here,  $\bar{V}$  is the average velocity,  $V_o$  is the centreline velocity, and  $n$  is a parameter. The ratio of average velocity to the centreline velocity for turbulent flow through the pipe is given by

$$\frac{\bar{V}}{V_o} = \frac{2n^2}{(n+1)(2n+1)}$$

For  $n = 7$ , the value of  $\frac{\alpha_{turbulent}}{\alpha_{laminar}}$  is \_\_\_\_\_ (round off to 2 decimal places).

Ans. 0.52 – 0.54

Exp:

Kinetic energy correction factor ( $\alpha$ ) Ratio for Laminar & Turbulent

$\alpha$  (KE correction factor) = 2 for Laminar &

$\alpha = (1.01 \text{ to } 1.15)$  for turbulent if we take  $\alpha = 1$  (for Turbulent)

$$\text{then } \frac{(\alpha)_{laminar}}{(\alpha)_{turb}} = \frac{2}{1} = 2$$

$$\alpha_t = \left(\frac{8 \times 15}{49 \times 2}\right)^3 \left(\frac{49 \times 2}{10 \times 17}\right) = 1.05$$

$$\alpha_L = 2$$

$$\beta = \frac{4}{3} = 1.33$$

$$\frac{\alpha_t}{\alpha_L} = \frac{1.05}{2} = 0.525 \approx 0.53$$

23. The molar heat capacity at constant pressure  $C_p$  (in  $\text{J mol}^{-1} \text{K}^{-1}$ ) for n-pentane as a function of temperature (T in K) is given by

$$\frac{C_p}{R} = 2.46 + 45.4 \times 10^{-3} T - 14.1 \times 10^{-6} T^2, \text{ Take } R = 8.314 \text{ J mol}^{-1} \text{K}^{-1}.$$

At 1000 K, the rate of change of molar entropy of n-pentane with respect to temperature at constant pressure is \_\_\_\_\_  $\text{J mol}^{-1} \text{K}^{-2}$  (round off to 2 decimal places).

Ans. 0.27 – 0.29

Exp:

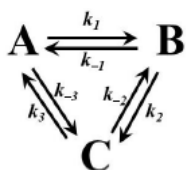
We know that

$$ds = \frac{dQ}{T} = \frac{mcpdT}{T}$$

$$\frac{ds}{dT} = \frac{mcp}{T} = \frac{(a+bT+cT^2+dT^3)R}{T}$$

On putting values, you will get the answer.

24. The following homogeneous liquid phase reactions are at equilibrium.



The values of rate constants are given by :

$$k_1 = 0.1 \text{ s}^{-1}, k_{-1} = 0.2 \text{ s}^{-1}, k_2 = 1 \text{ s}^{-1}, k_{-2} = 10 \text{ s}^{-1}, k_3 = 10 \text{ s}^{-1}$$

The value of rate constant  $k_{-3}$  is \_\_\_\_\_  $\text{s}^{-1}$  (correct to 1 decimal place).

Ans. 0.5

Exp:

∴ All the reactions are in equilibrium,

Therefore,

$$k_1 \cdot k_2 \cdot k_3 = 1$$

$$\Rightarrow \frac{k_1}{k_{-1}} \times \frac{k_2}{k_{-2}} \times \frac{k_3}{k_{-3}} = 1$$

$$\Rightarrow \frac{0.1}{0.2} \times \frac{1}{10} \times \frac{10}{k_{-3}} = 1$$

$$\Rightarrow k_{-3} = 0.5$$

25. A company invests in a recovery unit to separate valuable metals from effluent streams. The total initial capital investment of this unit is Rs. 10 lakhs. The recovered metals are worth Rs. 4 lakhs per year.

If the annual return on this investment is 15% , the annual operating costs should be \_\_\_\_\_ lakhs of rupees (correct to 1 decimal place).

Ans. 2.5

Exp:

FCI → 10 Lakhs

Annual return = 15%

Profit = 4 Lakhs/year

$$\text{ROR} = \frac{\text{PBT}}{\text{FCI}}$$

$$0.15 = \frac{\text{PBT}}{\text{FCI}}$$

$$0.15 \times 10 = \text{PBT}$$

$$1.5 \text{ lakhs} = \text{PBT}$$

PBT = Revenue – expenses

$$1.5 = 4 - \text{Annual operating cost}$$

$$\text{Annual operating cost} = 2.5 \text{ Lakhs}$$

**Q.26 - Q.33 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: - 2/3).**

26. Let A be a square matrix of size  $n \times n$  ( $n > 1$ ). The element of  $A = \{a_{ij}\}$  are given by

$$a_{ij} = \begin{cases} i \times j, & \text{if } i \geq j \\ 0, & \text{if } i < j \end{cases}$$

The determinant of A is

- (a) 1
- (b) 1
- (c) n!
- (d) (n!)<sup>2</sup>

Ans. d

Exp:

Let n = 2,

$$\begin{matrix} & i(1) & j(2) \\ i(1) & \begin{bmatrix} 1 & 0 \end{bmatrix} \\ j(2) & \begin{bmatrix} 0 & 4 \end{bmatrix} \end{matrix}$$

$$|A| = 4 = (2)^2 \rightarrow (n!)^2$$

27. Consider a fluid confined between two horizontal parallel plates and subjected to shear flow. In the first experiment, the plates are separated by a distance of 1 mm. It is found that a shear stress of 2 Nm<sup>-2</sup> has to be applied to keep the top plate moving with a velocity of 2 m/s, while the other plate is fixed.

In the second experiment, the plates are separated by a distance of 0.25 mm. It is found that a shear stress of 3 Nm<sup>-2</sup> has to be applied to keep the top plate moving with a velocity of 1 m/s<sup>-1</sup>, while the other plate is fixed.

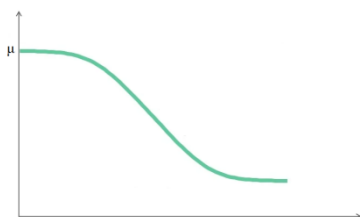
In the range of shear rates studied, the rheological character of the fluid is

- (a) Newtonian
- (b) Pseudoplastic
- (c) Dilatant
- (d) Ideal and inviscid

Ans. b

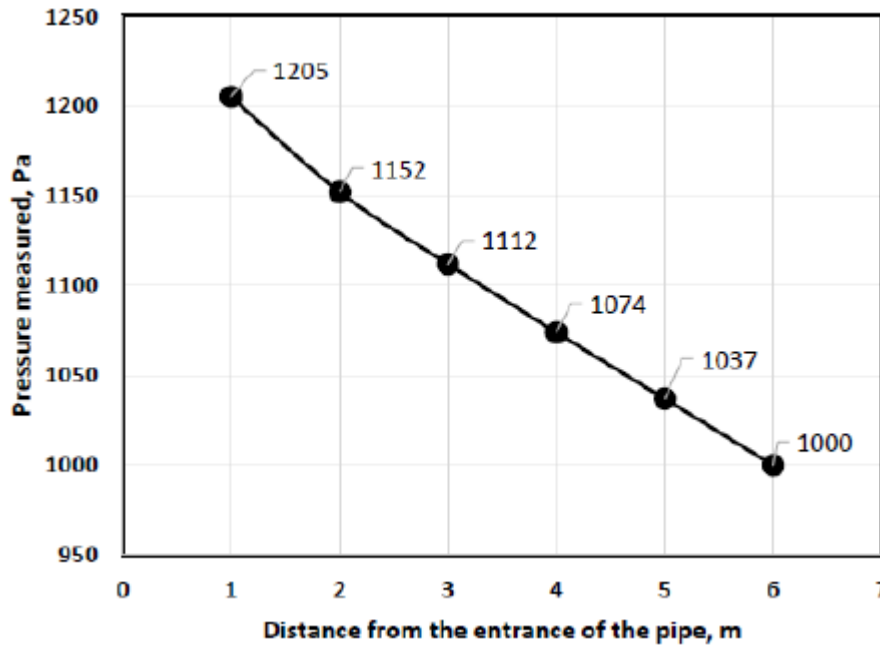
Exp:

Pseudoplastic



$$\left( \frac{d\theta}{dt} = \frac{du}{dy} \right)$$

28. Water of density  $1000 \text{ kg m}^{-3}$  flows in a horizontal pipe of 10 cm diameter at an average velocity of 0.5 m/s. The following plot shows the pressure measured at various distances from the pipe entrance.

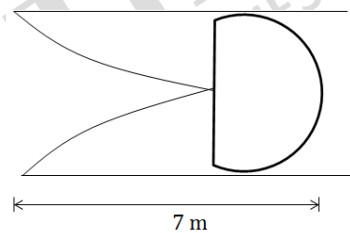


Using the data shown in the figure, the Fanning friction factor in the pipe when the flow is FULLY DEVELOPED is

- (a) 0.0012
- (b) 0.0074
- (c) 0.0082
- (d) 0.0106

Ans. b

Exp:



Fully developed,

$$\frac{\Delta P}{\rho g} = \frac{4C_f L V^2}{2gD}$$

$$C_f = \frac{\Delta P \cdot D}{2CV^2 \cdot \rho}$$

$$\Delta P = (1037 - 1000) Pa = 37 Pa$$

$$D = 0.1 \text{ m}$$

$$V = 0.5 \text{ m/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$Cf = \frac{37 \times 1}{2 \times 1 \times (0.5)^2 \times 1000} = 0.0074$$

29. In a solvent regeneration process, a gas is used to strip a solute from a liquid in a countercurrent packed tower operating under isothermal condition. Pure gas is used in this stripping operation. All solutions are dilute and Henry's law,  $y^* = mx$ , is applicable. Here  $y^*$  is the mole fraction of the solute in the gas phase in equilibrium with the liquid phase of solute mole fraction  $x$  and  $m$  is the Henry's law constant.

Let  $x_1$  be the mole fraction of the solute in the leaving liquid, and  $x_2$  be the mole fraction of solute in the entering liquid.

When the value of the ratio of the liquid-to-gas molar flow rates is equal to  $m$ , the overall liquid phase Number of Transfer Units,  $NTU_{OL}$  is given by

(a)  $\frac{x_2 - x_1}{x_1}$

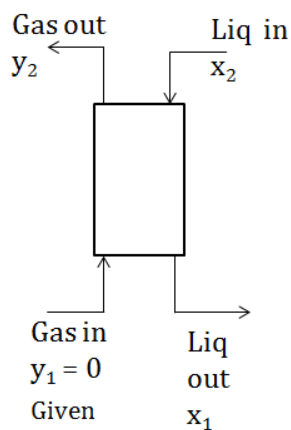
(b)  $\frac{x_2 + x_1}{x_2 - x_1}$

(c)  $\ln\left(\frac{x_2}{x_1}\right)$

(d)  $\ln\left(\frac{x_2 + x_1}{x_2 - x_1}\right)$

Ans. a

Exp:





For Absorption, we know that

$$N_{tOG} = \int_{y_2}^{y_1} \frac{(1-y)_{\ln}^*}{(1-y)(y-y^*)} dy$$

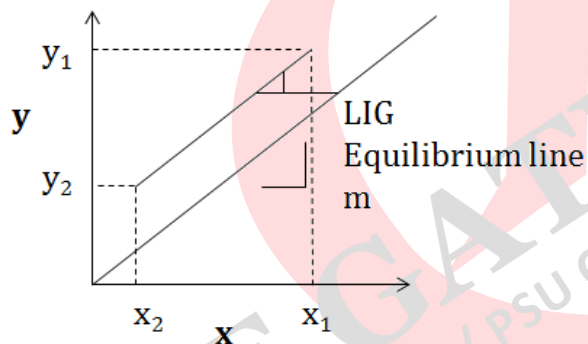
For diluted streams,  $1 - y \approx 1$

$$(1-y)_{\ln}^* \approx 1$$

$$\Rightarrow N_{tOG} \approx \int_{y_2}^{y_1} \frac{dy}{(y-y^*)}$$

As given,  $\frac{L}{G} = m$ , thus, the average driving force  $(y - y^*)$  will be same and constant throughout, therefore,

$$N_{tOG} \approx \frac{y_1 - y_2}{(y - y^*)}$$



Similarly, for stripping,

$$N_{tOL} = \int_{x_2}^{x_1} \frac{(1-x)_{\ln}^*}{(1-x)(x^*-x)} dx$$

For diluted streams,

$$N_{tOL} \approx \int_{x_2}^{x_1} \frac{dx}{(x^*-x)}$$

& for  $L/G = m$

$$N_{tOL} = \frac{x_1 - x_2}{(x^* - x)}$$

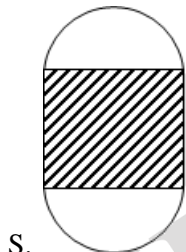
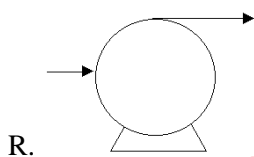
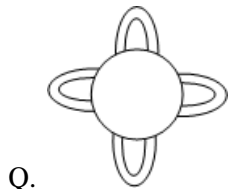
Since,  $(x^* - x)$  is average driving force which will be same and constant throughout, thus,

$$x^* - x = x_1^* - x_1 = x_2^* - x_2$$

$$x_1^* - x_1 \Rightarrow x_1^* = \frac{y_1}{m} = 0 \text{ as } y_1 = 0$$

$$\text{Therefore, } N_{ioL} = \frac{x_1 - x_2}{(-x_1)} = \frac{x_2 - x_1}{x_1}$$

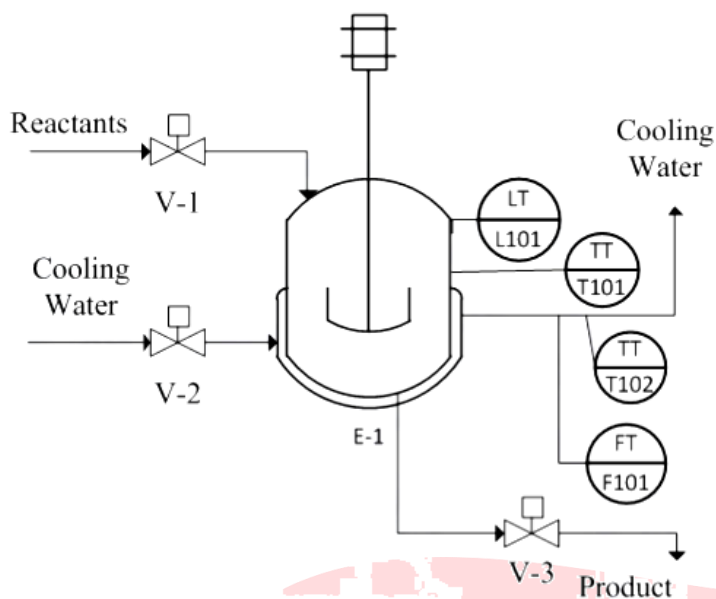
30. Which of these symbols can be found in piping and instrumentation diagrams ?



- (a) (Q) and (S) only  
 (b) (P), (Q) and (R) only  
 (c) (P), (R) and (S) only  
 (d) (P), (Q), (R) and (S)

Ans. c

31. It is required to control the volume of the contents in the jacketed reactor shown in the figure.



Which one of the following schemes can be used for feedback control?

- (a) Measure L101 and manipulate valve V-2
- (b) Measure T101 and manipulate valve V-1
- (c) Measure L101 and manipulate valve V-3
- (d) Measure F101 and manipulate valve V-1

Ans. c

32. Which of the following is NOT a necessary condition for a process under closed-loop control to be stable?

- (a) Dead-time term(s) must be absent in the open-loop transfer function
- (b) Roots of the characteristic equation must have negative real part
- (c) All the elements in the left (first) column of the Routh array must have the same sign
- (d) Open-loop transfer function must have an amplitude ratio less than 1 at the critical frequency

Ans. a

33. Match the reaction in Group – 1 with the reaction type in Group – 2.

Column – I	Column – II
P. Methyl cyclohexane $\rightarrow$ Toluene + 3H <sub>2</sub>	1. (Dehydrocyclization)
Q. Ethyl cyclo pentane $\rightarrow$ Methyl cyclo hexane	2. (Cracking )
R. n-Octane $\rightarrow$ Ethylbenzene + 4H <sub>2</sub>	3. (Dehydrogenation)

S. n-Octane $\rightarrow$ n-pentane + Propylene	4. (Isomerization)
---	--------------------

The correct combination is :

- (a) P - II, Q - III, R - I, S - IV
- (b) P - III, Q - IV, R - I, S - II
- (c) P - III, Q - IV, R - II, S - I
- (d) P - I, Q - IV, R - III, S - II

Ans. b

**Q.34 - Q.55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).**

34. To solve an algebraic equation  $f(x) = 0$ , an iterative scheme of the type

$$x_{n+1} = g(x_n) \text{ is proposed, where } g(x) = x - \frac{f(x)}{f'(x)}.$$

At the solution  $x = s$ ,  $g'(s) = 0$  and  $g''(s) \neq 0$ .

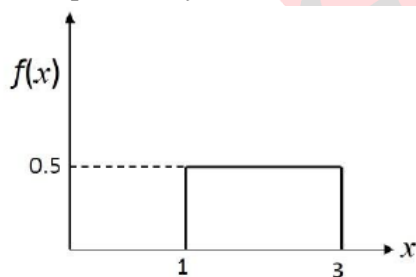
The order of convergence for this iterative scheme near the solution is \_\_\_\_\_.

Ans. 2

Exp:

Order of convergence of NR  $\rightarrow$  2

35. The probability distribution function of a random variable X is shown in the following figure.



From this distribution, random samples with sample size  $n = 68$  are taken. If  $\bar{X}$  is the sample mean, the standard deviation of the probability distribution of  $\bar{X}$ , i.e.,  $\sigma_{\bar{X}}$  is \_\_\_\_\_ (round off to 3 decimal places).

Ans. 0.069 - 0.071

Exp:

For continuous random variable,

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} (x - \mu)^2 P(x)}{n} \quad (i)$$

Here,  $\mu$  = expectation of  $x$ , and is given by

$$\mu = \int_{-\infty}^{\infty} (x) P(x)$$

$$\mu = \int_1^3 (x) 0.5 dx = 2$$

From (i)

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} (x - \mu)^2 P(x)}{68} = \frac{\int_{-1}^3 (x - 2)^2 0.5(x)}{68} = \frac{1}{3 \times 68} = \frac{1}{206} = 0.004854$$

$$\sigma = \sqrt{0.004854} = 0.0696$$

$$\text{Standard deviation} = \sqrt{\sigma^2}$$

$$P(X) = \begin{cases} 0 & , & x < 1 \\ 0.5 & , & 1 \leq x \leq 3 \\ 0 & , & x > 3 \end{cases}$$

36. For the ordinary differential equation

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 11 \frac{dy}{dt} + 6y = 1$$

With initial conditions  $y(0) = y'(0) = y''(0) = y'''(0) = 0$ , the value of  $\lim_{t \rightarrow \infty} y(t) = \underline{\hspace{2cm}}$  (round off to 3 decimal places).

Ans. 0.161 – 0.169

Exp:

Auxiliary Equation

$$m^3 + 6m^2 + 11m + 6 = 0$$

$$(m + 1)(m^2 + 5m + 6) = 0$$

$$(m + 1)(m + 2)(m + 3) = 0$$

$$CF = y = C_1 e^{-x} + e_2 e^{-2x} + C_3 e^{-3x}$$

$$PI = \frac{1}{D^3 + 6D^2 + 11D + 6} = \frac{1 \cdot e}{D^3 + 6D^2 + 11D + 6}$$

$$\Rightarrow PI = \frac{1 \cdot e^{0x}}{0^3 + 6 \times 0^2 + 11 \times 0 + 6} = \frac{1}{6}$$

Solution is given by

$$y = CF + PI$$

$$y = C_1 e^{-x} + c_2 e^{-2x} + e_3 e^{-3x} + \frac{1}{6}$$

At  $x = \infty$

$$y = C_1 e^{-\infty} + C_2 e^{-2 \times \infty} + C_3 e^{-3 \times \infty} + \frac{1}{6}$$

$$y = \frac{1}{6} = 0.166$$

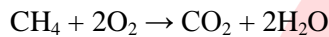
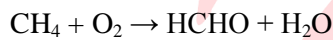
**Alternative Method:**

$$y = c_1 e^{-t} + c_2 e^{-2t} + c_3 e^{-3t} + \frac{1}{6}$$

At  $t = 0$

$$y = \frac{1}{6}$$

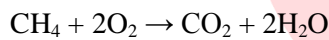
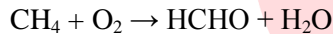
37. Formaldehyde is produced by the oxidation of methane in a reactor. The following two parallel reactions occur.



Methane and oxygen are fed to the reactor. The product gases leaving the reactor include methane, oxygen, formaldehyde, carbon dioxide and water vapour.

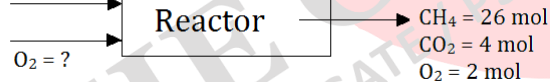
Ans. 40

Exp:



$$\text{CH}_4 = 60 \text{ mol}$$

$$\text{CH}_4 = 60 \text{ mol}$$



$\text{O}_2$  entered = ?

Sol.

Basis:

$$26 \frac{\text{mol}}{\text{s}} \text{CH}_4 + 4 \frac{\text{mol}}{\text{s}} \text{CO}_2 + 2 \frac{\text{mol}}{\text{s}} \text{O}_2 \text{ out}$$

From reaction 2,

$$1 \text{ mol CO}_2 \text{ required} = 1 \text{ mol CH}_4 \text{ \& } 2 \text{ mol O}_2$$

$$\text{So, } 4 \frac{\text{mol}}{\text{s}} \text{CO}_2 \text{ --- } = 4 \frac{\text{mol}}{\text{s}} \text{CH}_4 \text{ \& } 8 \frac{\text{mol}}{\text{s}} \text{O}_2$$

$$\text{CH}_4 \text{ reacted in rxn (1)} = 60 - 26 - 4 = 30 \frac{\text{mol}}{\text{s}}$$

From rxn (1),

$$\text{O}_2 \text{ reacted in reaction (1) with } 30 \frac{\text{mol}}{\text{s}} \text{ of CH}_4 = 30 \frac{\text{mol}}{\text{s}}$$

So,  $O_2 \text{ fed} = 30 + 8 + 2 = 40 \frac{\text{mol}}{\text{s}}$

38. The combustion of carbon monoxide is carried out in a closed, rigid and insulated vessel 1 mol of CO, 0.5 mol of  $O_2$  and 2 mol of  $N_2$  are taken initially at 1 bar and 298 K, and the combustion is carried out to completion.

The standard molar internal energy change of reaction ( $\Delta\mu_R^\circ$ ) for the combustion of carbon monoxide at 298 K =  $-282 \text{ kJ/mol}$ . At constant pressure, the molar heat capacities of  $N_2$  and  $CO_2$  are  $33.314 \text{ J mol}^{-1} \text{ K}^{-1}$  and  $58.314 \text{ J mol}^{-1} \text{ K}^{-1}$ , respectively. Assume the heat capacities to be independent of temperature, and the gases are ideal. Take  $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ .

The final pressure in the vessel at the completion of the reaction is \_\_\_\_\_ bar (round off to 1 decimal places).

Ans. 8.9 – 9.1

Exp:

As we know that,

$$\begin{aligned} \Delta H_{rxn}^\circ &= \Delta U_{rxn}^\circ + vRT \\ &= -282 \frac{\text{kJ}}{\text{mol}} + \left(-\frac{1}{2}\right) \times 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \times 298 \text{ K} \end{aligned}$$

$$\Delta H_{rxn}^\circ = -283238.786 \text{ J/mol}$$

& for the system, heat released = heat absorbed

$$\begin{aligned} \text{Thus, } [-\Delta H_{rxn}^\circ] &= \sum (n_i c_{pi}) \Delta T_{products} \\ \Rightarrow 1 \text{ mol} \times (+283238.786) \frac{\text{J}}{\text{mol}} &= (n_{CO_2} C_{pCO_2} + n_{N_2} C_{pN_2}) \Delta T \\ \Rightarrow +283238.786 \text{ J} &= [1 \times 58.314 + 2 \times 33.314] (T_2 - 298) \\ \Rightarrow T_2 &= 2564.96 \text{ K} \end{aligned}$$

And for constant volume system,

$$\begin{aligned} \frac{P_1}{n_1 T_1} &= \frac{P_2}{n_2 T_2} \\ \Rightarrow P_2 &= \frac{n_2 T_2}{n_1 T_1} P_1 \\ &= \frac{3 \text{ mol}}{3.5 \text{ mol}} \times \frac{2564.69 \text{ K}}{298 \text{ K}} \times 1 \text{ bar} \\ &= 7.38 \text{ bar} \approx 7.4 \text{ bar} \end{aligned}$$

39. A gaseous mixture at 1 bar and 300 K consists of 20 mol% CO<sub>2</sub> and 80 mol% inert gas. Assume the gases to be ideal. Take R = 8.314 J mol<sup>-1</sup> K<sup>-1</sup>.

The magnitude of minimum work required to separate 100 mol of this mixture at 1 bar and 300 K into pure CO<sub>2</sub> and inert gas at the same temperature and pressure is \_\_\_\_\_ kJ (round off to nearest integer).

Ans. 124 – 126

Exp:

At Constant T & p, the work required to separate a mixture in to its constituents will be exactly same in its magnitude as the work required for mixing of these components.

i.e.

$$\begin{aligned} \text{Work required for separation} &= - \text{Work required for mixing} \\ &= - \Delta G_{\text{mix}} \end{aligned}$$

$$\begin{aligned} \text{For ideal solution} &= -RT \sum x_i \ln x_i = -RT [x_1 \ln x_1 + x_2 \ln x_2] \\ &= -8.314 \times 300 \times [0.2 \times \ln 0.2 + 0.8 \times \ln 0.8] \\ &= 124810.37 \text{ J} \\ &= 124.81 \text{ kJ} \approx 125 \text{ kJ} \end{aligned}$$

40. A binary liquid mixture consists of two species 1 and 2. Let  $\gamma$  and  $x$  represent the activity coefficient and the mole fraction of the species, respectively. Using a molar excess Gibbs free energy model,  $\ln \gamma_1$  vs  $x_1$  and  $\ln \gamma_2$  vs  $x_1$  are plotted. A tangent drawn to the  $\ln \gamma_1$  vs  $x_1$  curve at a mole fraction of  $x_1 = 0.2$  has a slope = -1.728.

The slope of the tangent drawn to the  $\ln \gamma_2$  vs  $x_1$  curve at the same mole fraction is \_\_\_\_\_ (correct to 3 decimal places).

Ans. 0.432

Exp:

From Gibbs Duhem equation:

$$x_1 \frac{d \ln Y_1}{dx_1} = x_2 \frac{d \ln Y_2}{dx_2} = x_2 \frac{d \ln Y_1}{(-dx_1)}$$

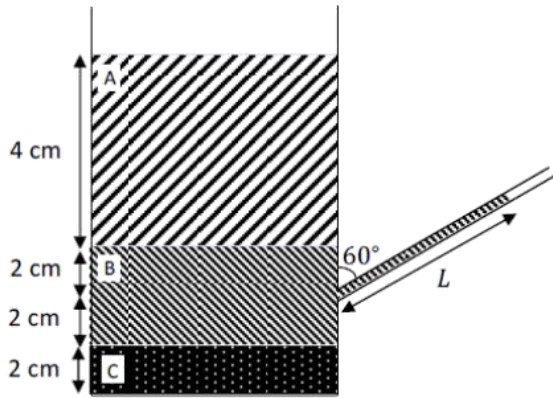
$$\Rightarrow \frac{d \ln Y_2}{dx_1} = \frac{-x_1}{x_2} \cdot \frac{d \ln Y_1}{dx_1}$$

$$= \frac{-0.2}{0.8} \times (-1.728)$$

$$\frac{d \ln Y_2}{dx_1} = 0.432$$

41. Consider a tank filled with 3 immiscible liquids A, B and C at static equilibrium, as shown in the figure. At 2 cm below the liquid A – liquid B interface, a tube is connected from the side of the tank. Both the tank and the tube are open to the atmosphere.

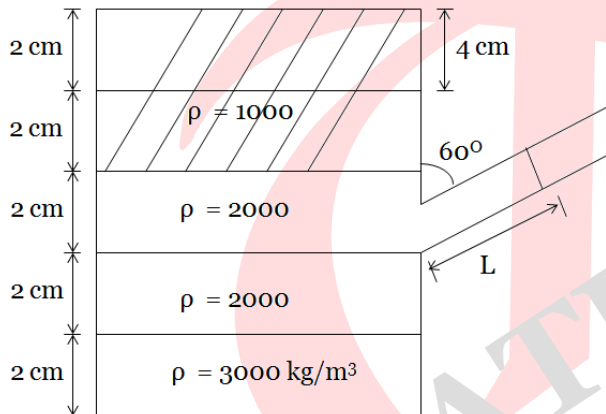




At the operating temperature and pressure, the specific gravities of liquids A, B and C are 1, 2 and 4 respectively. Neglect any surface tension effects in the calculations. The length of the tube L that is wetted by liquid B is \_\_\_\_\_ cm.

Ans. 8

Exp:



L (in cm) = ?

⇒ Applying Manometric Equation :

$$\frac{4}{100} \times g \times 1000 + \frac{2}{100} \times g \times 2000 = \frac{L \cos 60^\circ}{100} \times 2000 \times g$$

$$L = \frac{(4 \times 1000 + 2 \times 2000) \times 2}{2000} = 8 \text{ cm}$$

42. Seawater is passed through a column containing a bed of resin beads.

Density of seawater =  $1025 \text{ kg/m}^3$

Density of resin beads =  $1330 \text{ kg/m}^3$

Diameter of resin beads =  $50 \mu\text{m}$

Void fraction of the bed at the onset of fluidization = 0.4

Acceleration due to gravity =  $9.81 \text{ m/s}^2$

The pressure drop per unit length of the bed at the onset of fluidization is \_\_\_\_\_  $\text{Pa m}^{-1}$  (round off to nearest integer).

Ans. 1790 – 1800

Exp:

From Force Balance,

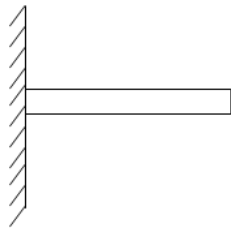
$$\frac{\Delta P}{L} = (\rho_p - \rho_f)(1 - \varepsilon)g$$

$$\frac{\Delta P}{L} = (1330 - 1025)(1 - 0.4) \times 9.81 = 1795.23 \frac{\text{Pa}}{\text{m}} \approx 1795$$

43. A straight fin of uniform circular cross section and adiabatic tip has an aspect ratio (length/diameter) of 4. If the Biot number (based on radius of the fin as the characteristic length) is 0.04, the fin efficiency is \_\_\_\_\_ % (round off to the nearest integer).

Ans. 42 – 44

Exp:



We know that, for fin efficiency,

$$\eta = \frac{\tanh mL}{mL}$$

where,  $mL = \sqrt{\frac{hp}{kA}} \cdot L^2$

As  $P = \pi d$

$$\text{Area} = \frac{\pi}{4} D^2$$

$$mL = \sqrt{\frac{h\pi D}{k \frac{\pi}{4} D^2}} \cdot L^2 = \sqrt{\frac{h \times 2 \times 4 \times R}{k} \left(\frac{L}{D}\right)^2} = \sqrt{8 \times 0.4 \times 16}$$

$$mL = 2.2627417$$

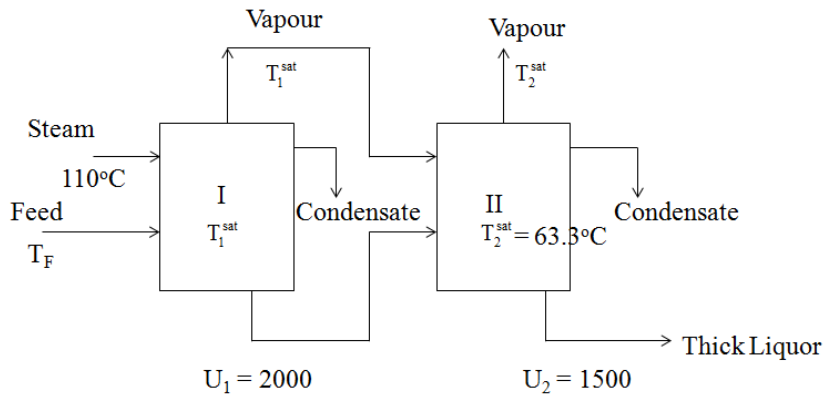
$$\text{Fin } \eta = 0.4324 \text{ i.e., } 43.24\%$$

44. A double-effect evaporator is used to concentrate a solution. Steam is sent to the first effect at 110°C and the boiling point of the solution in the second effect is 63.3°C. The overall heat transfer coefficient in the first effect and second effect are 2000 Wm<sup>-2</sup>K<sup>-1</sup> and 1500 Wm<sup>-2</sup>K<sup>-1</sup>, respectively. The heat required to raise the temperature of the feed to the boiling point can be neglected. The heat flux in the two evaporators can be assumed to be equal.

The temperature at which the solution boils in the first effect is \_\_\_\_\_ °C (round off to nearest integer).

Ans. 89 – 91

Exp:



The driving force in evaporators

$$\Delta T = T^{\text{steam}} - T^{\text{sat}}$$

As given heat load is same in both effects, i.e.,  $Q_1 = Q_2$

As  $A_1 = A_2$  rule of thumb

$$\Rightarrow U_1 A_1 \Delta T_1 = U_2 A_2 \Delta T_2$$

$$\Rightarrow U_1 (T_s - T_1^{\text{sat}}) = U_2 (T_1^{\text{sat}} - T_2^{\text{sat}})$$

$$\Rightarrow 2000(110 - T_1^{\text{sat}}) = 1500(T_1^{\text{sat}} - 63.3)$$

$$\Rightarrow T_1^{\text{sat}} \approx 90^\circ\text{C}$$

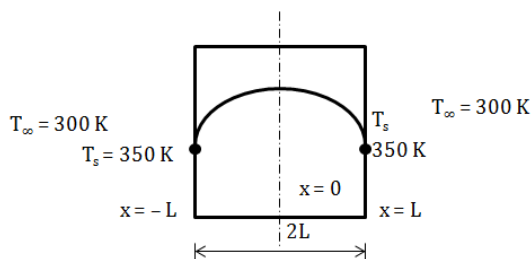
45. Consider a solid slab of thickness  $2L$  and uniform cross-section  $A$ . The volumetric rate of heat generation within the slab is  $\dot{g}$  ( $\text{Wm}^{-3}$ ). The slab loses heat by convection at both the ends to air with heat transfer coefficient  $h$ . Assuming steady state, one-dimensional heat transfer, the temperature profile within the slab along the thickness is given by:

$$T(x) = \frac{\dot{g}L^2}{2k} \left[ 1 - \left( \frac{x}{L} \right)^2 \right] + T_s \quad \text{for } -L \leq x \leq L$$

where  $k$  is the thermal conductivity of the slab and  $T_s$  is the surface temperature. If  $T_s = 350$  K, ambient air temperature  $T_\infty = 300$  K, and Biot number (based on  $L$  as the characteristic length) is 0.5, the maximum temperature in the slab is \_\_\_\_\_ K (round off to nearest integer).

Ans. 362 - 363

Exp:



$$T(x) = \frac{\dot{g}L^2}{2k} \left[ 1 - \left( \frac{x}{L} \right)^2 \right] + T_s$$

For maximum temperature,  $\frac{dT_x}{dx} = 0 = \frac{\dot{g}L^2}{2k} \cdot \left(\frac{-2x}{L^2}\right)$

$$\therefore x = 0$$

$$T_{\max} = \frac{\dot{g}L^2}{2k} + T_s$$

$$\dot{g} \times A \times 2L = 2hA(350 - 300)$$

$$\dot{g} = \frac{50h}{L}$$

$$T_{\max} = \frac{50h}{L} \cdot \frac{L^2}{2k} + 350$$

$$= 25 \frac{hL}{k} + 350$$

$$= 25 \times 0.5 + 350 = 362.5 \text{ K}$$

46. A distillation column handling a binary mixture of A and B is operating at total reflux. It has two ideal stages including the reboiler. The mole fraction of the more volatile component in the residue ( $x_w$ ) is 0.1. The average relative volatility  $\alpha_{AB}$  is 4. The mole fraction of A in the distillate ( $x_D$ ) is \_\_\_\_\_ (round off to 2 decimal places).

Ans. 0.63 – 0.65

Exp:

For minimum number of ideal stages at total reflux, we know that, from Fenske's Equation,

$$N_m + 1 = \frac{\ln \left[ \left( \frac{x_D (1 - x_w)}{x_w (1 - x_D)} \right) \right]}{\ln \alpha_{avg}}$$

$$\Rightarrow N_m + 1 = 2$$

$$2 = \frac{\ln \left[ \left( \frac{x_D \times (1 - 0.1)}{0.1(1 - x_D)} \right) \right]}{\ln(4)}$$

$$\Rightarrow x_D = 0.64$$

47. In a batch drying experiment, a solid with a critical moisture content of 0.2 kg H<sub>2</sub>O/kg dry solid is dried from an initial moisture content of 0.35 kg H<sub>2</sub>O/kg dry solid to a final moisture content of 0.1 kg H<sub>2</sub>O/kg dry solid in 5 h. In the constant rate regime, the rate of drying is 2 kg H<sub>2</sub>O/(m<sup>2</sup> · h).

The entire falling rate regime is assumed to be uniformly linear. The equilibrium moisture content is assumed to be zero.

The mass of the dry solid per unit area is \_\_\_\_\_ kg/m<sup>2</sup> (round off to nearest integer).

Ans. 34 – 35

Exp:

We know that,

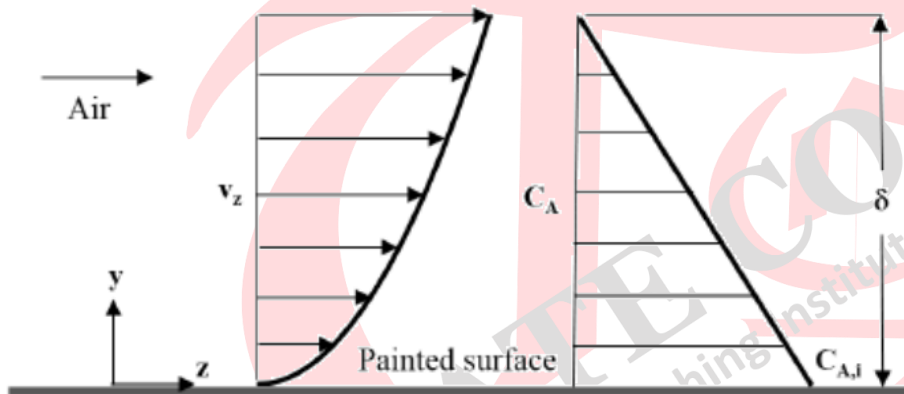
Total drying time,

$$t = t_c + t_F = \frac{W_s}{AR_c} \left[ (X_1 - X_c) + (X_c - X^*) \ln \left( \frac{X_c - X^*}{X_2 - X^*} \right) \right]$$

$$\Rightarrow 5 \text{ hr} = \frac{W_s}{A \times 2} \left[ (0.5 - 0.2) + (0.2 - 0) \ln \left( \frac{0.2 - 0}{0.1 - 0} \right) \right]$$

$$\Rightarrow \frac{W_s}{A} = 34.65 \approx 35 \frac{\text{kg}}{\text{m}^2}$$

48. As shown in the figure below, air flows in parallel to a freshly painted solid surface of width 10 m, along the z-direction.



The equilibrium vapour concentration of the volatile component A in the paint, at the air-paint interface, is  $C_{A,i}$ . The concentration  $C_A$  decreases linearly from this value to zero along the y-direction over a distance  $\delta$  of 0.1 m in the air phase. Over this distance, the average velocity of the air stream is 0.033 m/s and its velocity profile (in m/s) is given by

$$v_z(y) = 10y^2$$

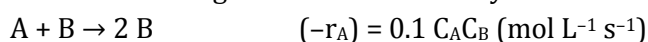
where y is in meter.

Let  $C_{A,m}$  represent the flow averaged concentration. The ratio of  $C_{A,m}$  to  $C_{A,i}$  is \_\_\_\_\_ (round off to 2 decimal places).

Ans. 0.24 – 0.26

Exp:

49. The following isothermal autocatalytic reaction



is carried out in an ideal continuous stirred tank reactor (CSTR) operating at steady state. Pure A at 1 mole L<sup>-1</sup> is fed, and 90% of A is converted in the CSTR. The space time of the CSTR is \_\_\_\_\_ seconds.

Ans. 100

Exp:

For CSTR,

We know that,

$$\tau = \frac{C_{A0} \cdot x_A}{(-r_A)} = \frac{C_{A0} \cdot x_A}{k C_A C_B}$$

where,  $C_A = C_{A0} (1 - x_A) = 1 \times (1 - 0.9) = 0.1 \text{ mol/l}$

$$C_B = C_{B0} + \frac{b}{a} C_{A0} \cdot x_A = 0 + \frac{1}{1} \times 0.9 = 0.9 \text{ mol/l}$$

On putting values,

$$\tau = \frac{1 \times 0.9}{0.1 \times 0.1 \times 0.9}$$

$$\Rightarrow \tau = 100 \text{ s}$$

50. Reactant A decomposes to products B and C in the presence of an enzyme in a well-stirred batch reactor. The kinetic rate expression is given by

$$-r_A = \frac{0.01 C_A}{0.05 + C_A} \text{ (mol L}^{-1} \text{ min}^{-1}\text{)}$$

If the initial concentration of A is  $0.02 \text{ mol L}^{-1}$ , the time taken to achieve 50% conversion of A is \_\_\_\_\_ min (round off to 2 decimal places).

Ans. 4.44 - 4.51

Exp:

We know that, for Batch Reactor,

$$t = \int_{-C_{A0}}^{C_A} \frac{dC_A}{(-r_A)} = \int_{C_A}^{C_{A0}} \frac{dC_A}{(-r_A)}$$

$$= \int_{C_A}^{C_{A0}} \frac{dC_A}{\frac{k_1 C_A}{k_2 + C_A}} = \int_{C_A}^{C_{A0}} \left( \frac{k_2 + C_A}{k_1 C_A} \right) dC_A$$

$$= \int_{C_A}^{C_{A0}} \left( \frac{k_2}{k_1 C_A} + \frac{1}{k_1} \right) dC_A$$

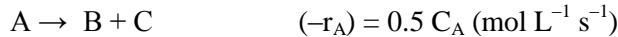
$$t = \frac{k_2}{k_1} \ln \left( \frac{C_{A0}}{C_A} \right) + \frac{1}{k_1} (C_{A0} - C_A)$$

As  $C_A = C_{A0} (1 - X_A) = 0.02 \times (1 - 0.5) = 0.01 \frac{\text{mol}}{\text{l}}$

So,  $t = \frac{0.05}{0.01} \ln \left( \frac{0.02}{0.01} \right) + \frac{1}{0.01} (0.02 - 0.01)$

= 4.466 min  $\approx$  4.47 min.

51. The following homogeneous, irreversible reaction involving ideal gases,



Is carried out in a steady state ideal plug flow reactor (PFR) operating at isothermal and isobaric conditions. The feed stream consists of pure A, entering at 2 m/s .

In order to achieve 50% conversion of A, the required length of the PFR is \_\_\_\_\_ meter (round off to 2 decimal places).

Ans. 3.49 – 3.61

Exp:

For PFR, we know that,

$$\tau = C_{A0} \int_0^{X_A} \frac{dX_A}{(-r_A)} = C_{A0} \int_0^{X_A} \frac{dX_A}{kC_A}$$

$$\tau = C_{A0} \int_0^{X_A} \frac{dX_A}{kC_{A0}(1-X_A)(1+\varepsilon_A X_A)}$$

$$\tau = \frac{1}{k} \int_0^{X_A} \frac{(1+\varepsilon_A X_A)}{(1-X_A)} dX_A$$

Here,  $\varepsilon_A = \delta \cdot y_{A0} = 1 \cdot 1 = 1$

$$\Rightarrow \tau = \frac{v}{v_0} = \frac{L}{u_0} = \frac{1}{k} \int_0^{0.5} \left( \frac{1+X_A}{1-X_A} \right) dX_A$$

L = 3.54 m

52. A system has a transfer function  $G(s) = \frac{3e^{-4s}}{12s+1}$ . When a step change of magnitude M is given to the system input, the final value of the system output is measured to be 120. The value of M is \_\_\_\_\_.

Ans. 40

Exp:

$$G(s) = \frac{3e^{-4s}}{12s+1}$$

$$\frac{\bar{Y}(s)}{\bar{X}(s)} = \frac{3e^{-4s}}{12s+1}$$

$$\bar{Y}(s) = \frac{m}{s} \times \frac{3e^{-4s}}{12s+1}$$

Final Value Theorem

$$Y(t) = sY(s)$$

$$\lim_{t \rightarrow \infty} \lim_{s \rightarrow 0}$$

$$\left( \bar{X}(s) = \frac{M}{s} \right)$$

Given  $\rightarrow$  Final value of response = 120

$$\bar{Y}(t) = s\bar{Y}(s) = 120$$

$$\lim_{t \rightarrow \infty} \lim_{t \rightarrow \infty}$$

$$\Rightarrow \lim_{s \rightarrow 0} s \times \frac{m}{s} \times \frac{3e^{-4s}}{12s+1} = 120$$

On putting limits

$$3M = 120$$

$$M = 40$$

53. A process has a transfer function  $G(s) = \frac{Y(s)}{X(s)} = \frac{20}{90,000s^2 + 240s + 1}$

Initially, the process is at steady state with  $x(t=0) = 0.4$  and  $y(t=0) = 100$ . If a step change in  $x$  is given from 0.4 to 0.5, the maximum value of  $y$  that will be observed before it reaches the new steady state is \_\_\_\_\_ (round off to 1 decimal place).

Ans. 102.4 – 102.6

Exp:

Given,  $G(s) = \frac{Y(s)}{X(s)} = \frac{20}{90,000s^2 + 240s + 1}$  (i)

$X(t=0) = 0.4$ ,  $y(t=0) = 100$

Step change in  $(x) \rightarrow 0.4 - 0.5$

$\Rightarrow x(t) = 0.1$

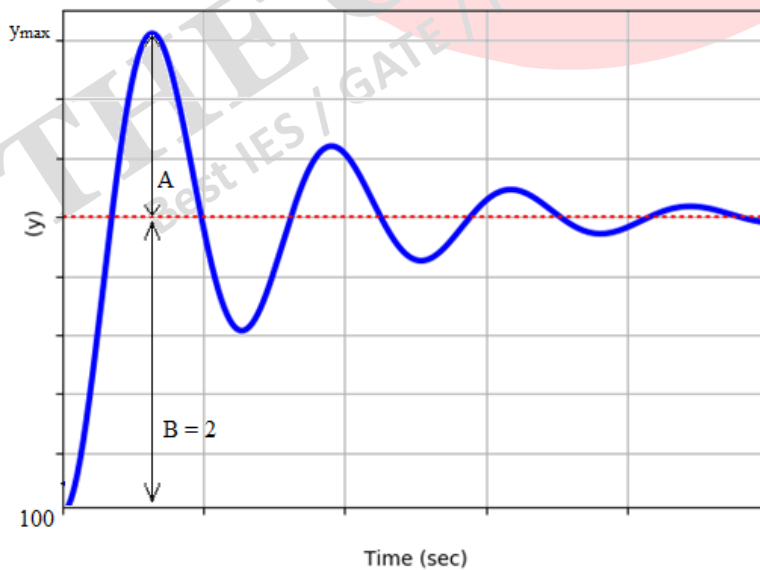
$x(s) = \frac{0.1}{s}$

We know that

$G(s) = \frac{Y(s)}{X(s)} = \frac{k_p}{\tau_p^2 s^2 + 2r\tau + 1}$  (ii)

From (i) and (ii),

$\tau = 300$  and  $r = 0.4$



Overshoot  $\Rightarrow \frac{A}{B} = \exp\left(\frac{-\pi r}{\sqrt{1-r^2}}\right)$



$$\frac{A}{B} = 0.2538$$

$$\Rightarrow Y_{\max} = (A + B)$$

$$Y_{\max} = B \left( \frac{A}{B} + 1 \right)$$

$$Y_{\max} = 2(0.2538 + 1)$$

$$Y_{\max} = 2.5076$$

$$Y_{\max} = y_{\max} - y_{ss}$$

$$2.5076 = y_{\max} - 100$$

$$y_{\max} = 102.5076 = 102.5$$

54. Operating labour requirements  $L$  in the chemical process industry is described in terms of the plant capacity  $C$  ( $\text{kg day}^{-1}$ ) over a wide range ( $10^3 - 10^6$ ) by a power law relationship

$L = \alpha C^\beta$  where  $\alpha$  and  $\beta$  are constants. It is known that

$L$  is 60 when  $C$  is  $2 \times 10^4$

$L$  is 70 when  $C$  is  $6 \times 10^4$

The value of  $L$  when  $C$  is  $10^5 \text{ kg day}^{-1}$  is \_\_\_\_\_ (round off to nearest integer).

Ans. 73 - 77

Exp:

$$L = \alpha C^\beta$$

$L$  is 60 when  $C$  is  $2 \times 10^4$

$L$  is 70 when  $C$  is  $6 \times 10^4$

$L \rightarrow ?$  when  $C$  is  $10^5$

$$\Rightarrow \frac{60}{70} = \frac{\alpha (2 \times 10^4)^\beta}{\alpha (6 \times 10^4)^\beta}$$

$$\Rightarrow \frac{6}{7} = \left( \frac{1}{3} \right)^\beta$$

Taking  $\ln$  on both sides, we have

$$\beta = 0.140313$$

$$60 = \alpha (2 \times 10^4)^{0.140313}$$

$$\alpha = 14.9506$$

$$L = 14.9506 [10^5]^{0.140313}$$

$$L = 75.201 \approx 75$$

55. A viscous liquid is pumped through a pipe network in a chemical plant. The annual pumping cost per unit length of pipe is given by

$$C_{pump} = \frac{48.13q^2\mu}{D^4}$$

The annual cost of the installed piping system per unit length of pipe is given by

$$C_{piping} = 45.92 D$$

Here,  $D$  is the inner diameter of the pipe in meter,  $q$  is the volumetric flowrate of the liquid in  $m^3/s$  and  $\mu$  is the viscosity of the liquid in Pa.s.

If the viscosity of the liquid is  $20 \times 10^{-3}$  Pa.s and the volumetric flow rate of the liquid is  $10^{-4} m^3/s$ , the economic inner diameter of the pipe is \_\_\_\_\_ m (round off to 3 decimal places).

Ans. 0.014 – 0.016

Exp:

Optimization

Optimize the diameter

$$C_{pumping} \rightarrow \frac{48.13q^2\mu}{D^4}, \quad C_{piping} = 45.92 D$$

$$\mu = 20 \times 10^{-3}$$

$$q = 10^{-4}$$

$$C_T \text{ (Total cost)} = \frac{48.13q^2\mu}{D^4} + 45.92D$$

$$\frac{d(C_T)}{dD} = \frac{(48.13q^2\mu)(-4)}{D^5} + 45.92$$

$$\text{For optimization} = \frac{48.13q^2\mu \times 4}{D^5} = 45.92$$

$$\frac{48.13 \times 10^{-8} \times 20 \times 10^{-3} \times 4}{45.92} = D^5$$

$$D = 0.0153 = 0.015$$